Including Uncertainties in Line Planning and Timetabling

Viera Klasovitá

Francesco Corman

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Viera Klasovitá
IVT
ETH Zurich
viera.klasovita@ivt.baug.ethz.ch

Francesco Corman
IVT
ETH Zurich
francesco.corman@ivt.baug.ethz.ch

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Abstract

This paper examines some important aspects of uncertainties in the line planning problem and in timetabling within the context of public transport. A literature review considers various methods for modelling the problem and associated uncertainties. These models range from (light) robustness to stochastic programming, e.g. 2-stage models, for stochastic parameters. In addition, the paper explores various solution strategies that have been applied to address uncertainties in line planning and related problems. The synthesis of the different literature papers shows which uncertainties are usually considered, which seem to be omitted, and different ways of modelling them. Moreover, it also reveals variations in terminology across different papers, highlighting the complexities researchers face.

Keywords

Line planning, Timetabling, uncertainty, multi-stage approaches, robustness, stochastic programming

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1 Introduction

In this paper, we examine different articles that consider public transport design. To be more precise, we consider network design, line planning and timetabling to be our scope. These three problems being the first steps in public transport design (Lusby *et al.*, 2018), they fix a lot for the other planning steps and therefore are essential regardless of the vehicle technology used, whether it is railway, tram, bus, other vehicle types, or any combination. Together, these problems prescribe already a lot of important factors like direct connections, travel time and frequency of service, which all significantly affect the mode choice and route choice of passengers.

In reality, the precise demand or travel times are not known and also change from day to day. Therefore, considering variable parameters is a crucial aspect of an effective public transport operation, a topic that many timetabling and line planning approaches still ignore and/or assume is solved at downstream stages. Only a few approaches explicitly consider this problem, employing reactive systems that are able to deal inherently with dynamic demand. In this paper, we focus on these papers in particular, where some inputs are not given as deterministic values.

This paper is organised as follows. In Section 2, we discuss the problems we considered when searching for papers. Then, in Section 3, we classify the models into three groups. In Section 4, we show the different approaches used to solve the problems at hand. Finally, in Section 5, we conclude with a brief discussion.

2 Problems considered

To get an overview of the different papers in this field, there were four aspects we considered to see the similarities and differences: the problem to solve, what kind of model used, which parameters are uncertain and what methods were used to solve the problem. Table 1 in Appendix A shows an overview of the different articles.
Getting a nice overview proved quite difficult in some of the areas, especially when defining the problem that is considered in the paper. This is due to the fact that some authors use different terms. A finished line plan for periodic setting should, according to Schöbel (2012), have a set of lines and the corresponding frequencies of these lines. Yet, a lot of network-design papers have the same outcome, while some papers talking

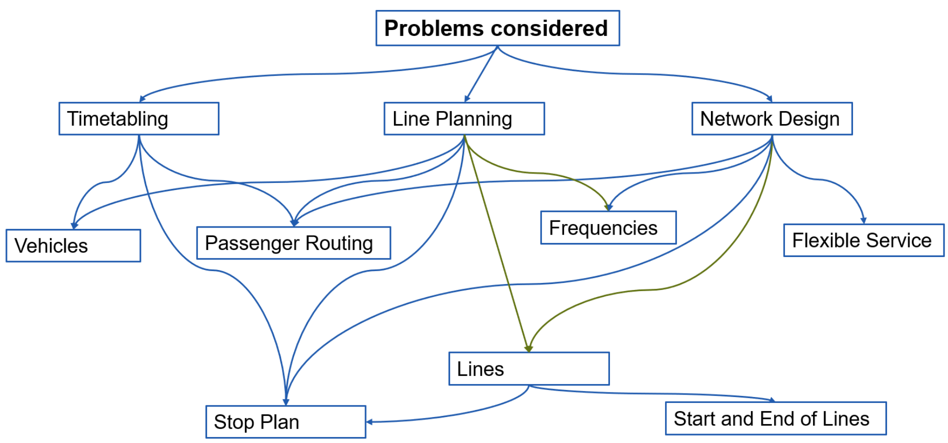
about line planning solve only possible subproblems. By using keywords as "choosing" or "determining" we try to be as precise as possible with the description of what the papers solve for. As a next point, we consider what kind of model is used. The papers considered using some kind of mathematical programming to describe constraints and solutions. Yet it is interesting to see which papers use linear programming, which use integer or binary programming, and also how the uncertainty is included - e.g. being a 2-stage model or using robustness.

Then, we wanted to compare the uncertainties. So, which parameters are taken as variables, whereas the others will be assumed to be fixed? With this, we also see how these models integrate stochasticity - is it through a protection level, which we often see for robust approaches, or is it, e.g., different scenarios for a stochastic programming approach?

Finally, we compare how the models solve their problems. Although there exist special cases and subproblems that can be solved in polynomial time, in general, the problems considered, that being Network Design, Line Planning Problem and Timetabling, are NP-hard (Guihaire and Hao, 2008), (Schöbel, 2012), (Caimi *et al.*, 2017), and therefore hard to solve. Therefore it is interesting to see which algorithms work well for solving the problems.

2.1 Poblems considered

Figure 1: Problems considered

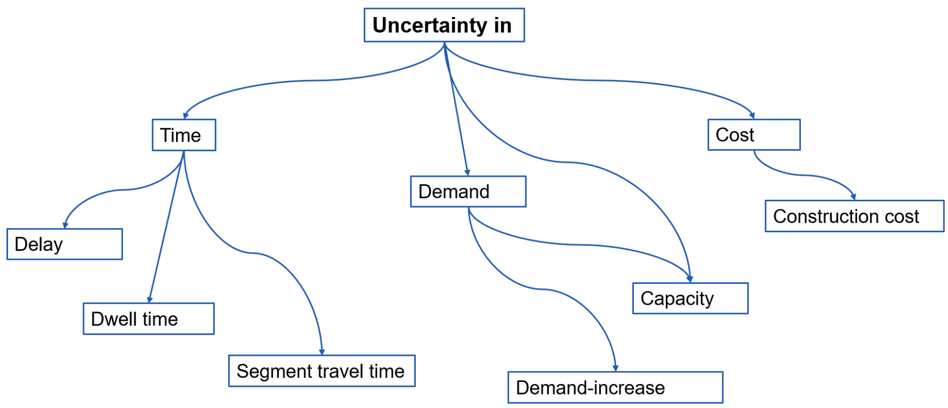


In general, we consider three problems in this literature review paper: network design problem, line planning problem, and timetabling. At first glance, these three problems follow each other, for example, in the railway planning process Lusby *et al.* (2018). However, looking at the papers in more detail, we can see that these problems are much more interconnected than first assumed. We try to show these connections with Figure 1. We

can see that many subproblems can be considered not only in one of the planning steps but in multiple. For example, routing passengers is something that is often done in all three steps to get a passenger-oriented solution. We can also see that network design and line planning share a lot of common possible subproblems, such as getting a line plan, determining frequencies or computing the stop plan. While not all papers consider all subproblems, they occur often in both steps.
A bit more different is the third step, computing a timetable. In this step, we want to assign arrival and departure times to the individual trains and route them through the network. However, also, this step can have common subproblems, such as routing of passengers or computing the stop plan.
We can see that these three problems are very interconnected and share some similar goals. As the solution of one step directly influences the solution of the next step, some papers also consider an integrated approach that considers multiple steps simultaneously.

2.2 Uncertain Parameters

Figure 2: Uncertain Parameters

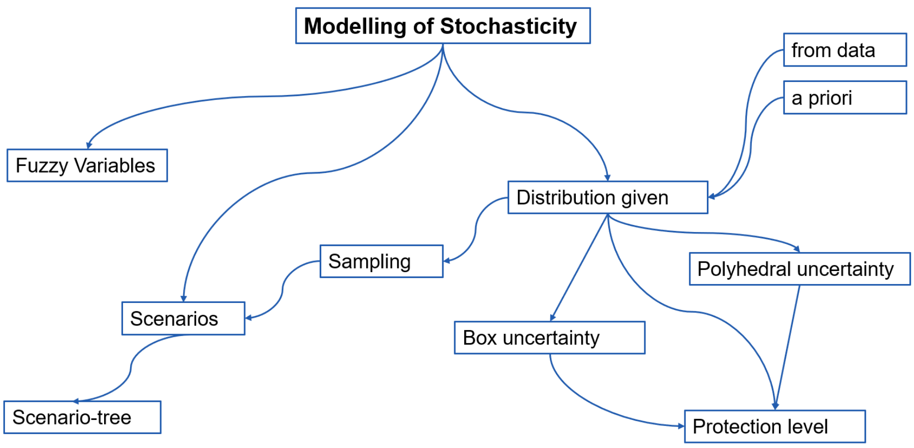


To see the differences and similarities of the papers, we can also consider the parameters that are considered uncertain. These can be split into three main groups: time, demand/-passengers, and cost.
Time uncertainty occurred most in timetabling. When considering an uncertainty in time, we saw that the papers considered various things. Some papers consider uncertainty in delays, which they usually want to protect themselves against. One can also consider that the dwell time and segment travel time are uncertain.
For line planning and network design, the uncertainty is usually caused by passengers

or cost. The number of passengers can increase or decrease and can have a significant influence on the network. An increase in demand could also lead to capacity problems, which some papers consider directly. Lastly, the cost of the solution can be considered. For line planning, that would usually be the operational cost of the vehicles, while in network design, the problem is typically the construction cost, which cannot be estimated precisely in advance.

2.3 Modelling of Stochasticity

Figure 3: Modelling Stochasticity



It is also interesting to see how the various papers model the uncertainty. Most papers work with either some distribution/protection level or with a set of scenarios. There are papers with other approaches, as using fuzzy variables (Yang *et al.*, 2009).
The distribution can either be given a prior or from data. Some articles mention that this distribution can be estimated from historical data (Shakibayifar *et al.*, 2017), but most articles do not consider the data behind it. Through sampling, the models can be reduced into problems using scenarios. There are also papers that directly work with scenarios.
Most distributions considered were given by a uniform distribution, where the authors considered a box uncertainty. This was especially the case for the papers doing robust optimisation, where the important part was the protection level given around some estimate. There were also papers considering other kinds of distribution, such as polyhedral uncertainty.

3 Classification of Modelling-Approaches

When dealing with uncertainties in the input parameter that solve any kind of problem, we can ask ourselves how to model this uncertainty. This is the first step of the process - if we do not have a model, we cannot really solve it. When modelling the uncertainty, it is important to think about why we are incorporating an uncertain parameter. Reading the papers, we have found that there are three main streams that incorporate the variability of some parameters.

- One way is to assume the parameters are known and deterministic but change over time.
- Another big stream is robustness. While there are different kinds of approaches to robustness (Lusby *et al.*, 2018), the common goal is to protect oneself from unpleasant situations, e.g. delays in timetables.
- Finally, one can use stochastic programming to incorporate the shape of the uncertainty into the problem.

There are papers with other modelling approaches, such as fuzzy mathematics, which we will touch upon shortly.

3.1 Variable Parameters

Solving real-life problems is difficult, as the input parameters are usually unknown and constantly changing. One approach is to consider the parameters to be given, but they fluctuate over time. We can assume the parameter changes in every time step, e.g. every minute. Hao *et al.* (2023) determine the lines, passenger routing and a timetable, where the demand is given for every minute. For every departure time, the passenger demands are given.

To simplify this, we can group the time steps together. Together, we now get periods, which is similar to considering individual time steps but just less detailed. Nie *et al.* (2023) look at line planning for the entire week. They introduce three different types of lines for different periods and days. They create a weekly line plan matching the passenger demand for the different periods. Şahin *et al.* (2020), Schiewe *et al.* (2023) and Zhou *et al.* (2023) all also consider a multi-period setting. While Şahin *et al.* (2020) and Schiewe *et al.* (2023) consider the problem of choosing lines, Zhou *et al.* (2023) also integrates timetabling into the problem.

3.2 Robustness

Another approach to modelling the uncertainty inputs is robustness. In the context of railway, a system is robust if it can continue its service at some level when faced with disruptions (Lusby *et al.*, 2018). This can be achieved by implementing a protection level or using light robustness to minimise the number of changes necessary to get a good system. Schöbel (2014) discusses a few different approaches to robustness.

Strict Robustness
As is explained in (Schöbel, 2014), when we deal with strict robustness, we want all possible values of the uncertain parameter to be feasible. Pu and Zhan (2021) consider passenger demand that is in a given box-set for which they want to find a robust solution. An and Lo (2015) and Wang *et al.* (2023) both consider a box-set, too, but they extend their model with the idea of Γ-robustness, i.e. giving an upper bound on the change possible (Bertsimas and Sim, 2003).

Light Robustness
Light robustness can be considered as the "flexible counterpart" to robust optimisation, as Fischetti and Monaci (2009) put it. Both Cacchiani *et al.* (2020) and Qi *et al.* (2018) consider light robustness in their approach. Cacchiani *et al.* (2020) solve the problem of timetabling, including considering the stop plan of the lines and passenger routing. They introduce different robust models, but they all want to protect against demand increase. Qi *et al.* (2018) solve the same problem, also protecting against a possible increase in demand. In their case, they consider the protection level, i.e. demand-increase, for every od-pair, with the protection level being limited by the residual capacity available.

3.3 Stochastic Programming

Another common approach is to use stochastic programming to solve the problem. When we introduce uncertainty in an input parameter, there is no such thing as an optimal solution. So, in stochastic optimisation, we usually want to find a good overall solution. This solution will hardly ever be optimal after the uncertain variable is realised, but it should produce a good solution for almost all cases. For this, we usually look at the deterministic equivalent of a stochastic problem by introducing the expectation of the objective and optimising this expectation (Kall and Wallace, 1994).
When using stochastic programming, we assume that we know the distribution of the random variable. There are two streams: some papers consider pre-defined scenarios or

by assuming probability distribution for the random variable.

Stochastic Programming with pre-defined Scenarios
Feng *et al.* (2023) consider the timetable problem with some additional subproblems. They deal with uncertain passenger demand given as a set of s stochastic scenarios. Khan and Zhou (2010) also consider the timetable problem with a number of scenarios. They, however, in contrast to (Feng *et al.*, 2023) consider the uncertainty in the segment travel time and in the departure times.
Cadarso *et al.* (2018) also use stochastic programming to determine lines and passenger flow. They introduce a scenario tree to get a discrete number of scenarios.

Stochastic Programming with Probability Distribution
An and Lo (2016) calculate lines, frequencies and passenger routing under uncertain demand. They sample a continuous distribution to get scenarios. Similarly, Kroon *et al.* (2008) also start with a random variable that is sampled. They do timetabling based on an input timetable and only change it slightly. They consider uncertainty in disturbances, looking at independent realisations of the timetable subject to a stochastic primary disturbance. While Shakibayifar *et al.* (2017) also consider the problem of generating a timetable, they consider the passenger arrival rates to the stations as stochastic. They assume that a probability distribution can be estimated from historical data. This distribution is then used to sample and introduce different scenarios.
Similarly to (Kroon *et al.*, 2008), Yin *et al.* (2016) also do timetabling based on an input timetable. They consider the passenger demand to be dynamic, where the arrival of passengers at the stations is a stochastic process. Yang *et al.* (2017) also consider timetabling (and calculations of the speed profile). They consider the dwell time to be a random variable with a given probability density function.

3.4 Other Approaches

Han and Ren (2020) consider the problem of stop planning and ticket allocation. They assume the passenger demand to be uncertain. They use uncertainty theory to transform the model to a deterministic one. Another approach is taken by Yang *et al.* (2009), where they solve a timetable. They consider the passenger demand as uncertain and model it as a fuzzy variable. And Yao *et al.* (2014) calculate lines and consider passenger demand. They assume the travel time is uncertain but use sampling to get a precise value used in a deterministic mathematical programming model.

4 Approaches to solve the problems

4.1 Variable Parameters

Solvers

While this might seem like a weird approach to state, just using a solver might be able to solve the problem for a certain size. Both Şahin *et al.* (2020) and Schiewe *et al.* (2023) do not introduce an algorithm and just use the Gurobi-solver for their problem.

Heuristics

The other approach taken in the mentioned papers is heuristics. Nie *et al.* (2023) solve line planning by implementing a genetic algorithm to solve their model. Zhou *et al.* (2023) use a simulated annealing algorithm to solve their problem. Moreover, they compare this to a particle swarm optimisation algorithm to see which algorithm works better for the problem considered. Hao *et al.* (2023) solve the same problem and also use a simulated annealing algorithm - in their case, a double-layer simulated annealing algorithm.

4.2 Robustness

Although we distinguished between strict and light robustness in the last section, the approaches to both are similar. Therefore, we do not distinguish the models here.

Solvers

As with the variable demand, there are papers that do not consider a separate algorithm for solving the problem. To solve timetabling Qi *et al.* (2018) use CPLEX. Cacchiani *et al.* (2020) investigate the same problem and also use CPLEX. Wang *et al.* (2023) use CPLEX to solve their problem of extending lines. To simplify it, they use a section-based strategy to decrease the number of constraints.

Algorithms

An and Lo (2015) use a cutting constraint algorithm, the Frank-Wolfe algorithm, to determine a line plan and solve the routing of passengers. Pu and Zhan (2021) use lagrangian relaxation to solve their two-stage model for stop planning and passenger routing.

4.3 Stochastic Programming

As with robustness, the approaches to solving the problem do not differ much, whether the model deals with individual scenarios or uses probability distributions.

Solvers

To solve timetabling Kroon *et al.* (2008) use CPLEX. Using the sample-average approximation method, they sample the distribution to get a deterministic model. Similarly, Shakibayifar *et al.* (2017) solve timetabling using CPLEX. Using the sample average approximation method, they compute lower and upper bounds, increasing the sample size until the estimated confidence intervals are close enough.

Algorithms

There are quite a few different approaches, in addition to the sample average approximation method, to obtain a solution to the problem. To solve the line planning problem An and Lo (2016) use a gradient method combined with neighbourhood search. Cadarso *et al.* (2018) consider a similar problem. They use a fix-and-relax and lazy metaheuristic algorithm with dynamic scenario (de)aggregation to solve the problem. Khan and Zhou (2010) consider the problem of obtaining a timetable. Under segment travel-time and departure time uncertainty, they use a heuristic sequential solution framework to get a timetable. Yin *et al.* (2016) use a approximate dynamic programming algorithm while Yang *et al.* (2017) use a genetic algorithm framework with ε-constrained method to obtain a solution. To solve timetabling, Feng *et al.* (2023) use a heuristic local search with branch-and-bound.

4.4 Other Approaches

Han and Ren (2020) use uncertainty theory to obtain a deterministic mathematical programming model. They use lagrangian relaxation heuristic algorithm to obtain a solution to their problem. Yang *et al.* (2009) introduced a fuzzy variable for passenger demand. To solve their model, they use a fuzzy simulation-based branch and bound. Yao *et al.* (2014) use tabu search to find a solution to determine lines and compute the passenger routing.

5 Conclusion

In conclusion, this paper provides a comprehensive overview of the research landscape surrounding uncertainty in network design, line planning, and timetabling within public transportation. With many recent studies, the paper offers valuable insights into the evolving discourse on this topic. Despite much recent research, there remains a limited exploitation of data, particularly in leveraging historical data for modelling the uncertainty. Having more information and data from the past could help to incorporate uncertainty in better detail in the model.

The consideration of robustness and stochasticity emerges as an important aspect in addressing uncertainties in planning processes, although their practical utility from both operator and passenger perspectives warrants further exploration. There are mathematical indicators, e.g., the value of stochastic solution, that can already imply that incorporating uncertainty can be useful. However, having more real-life test cases on which to test the models would be helpful. Moving forward, the establishment of common test cases could furthermore facilitate comparative analyses across modelling approaches, thereby fostering a deeper understanding of their strengths and limitations. Such common test cases would also help investigate the advantages and disadvantages of the different models further.

The confusion surrounding nomenclature, particularly regarding line planning and network design, highlights the importance of establishing standardised definitions and terminology in future research endeavours, such as the definition of the line planning problem proposed by Schöbel (2012). As the problems are similar, there is a lot that one can learn from each other: what kind of models exist and which algorithms are able to solve them.

In the future, it would be interesting to have a more detailed comparison of the approaches. Finding out whether robustness and what kind of stochastic programming is more efficient could help us design better public transport. Moreover, line planning usually focuses a lot on uncertainties in regard to demand. It would be interesting to also incorporate time-related uncertainties, for example, by taking a distribution on the travel time.

6 References

An, K. and H. K. Lo (2015) Robust transit network design with stochastic demand considering development density, *Transportation Research Part B: Methodological*, **81**, 737–754, ISSN 0191-2615. ISTTT 21 for the year 2015.

An, K. and H. K. Lo (2016) Two-phase stochastic program for transit network design under demand uncertainty, *Transportation Research Part B: Methodological*, **84**, 157–181, ISSN 0191-2615.

Bertsimas, D. and M. Sim (2003) Robust discrete optimization and network flows, *Mathematical programming*, **98** (1) 49–71.

Cacchiani, V., J. Qi and L. Yang (2020) Robust optimization models for integrated train stop planning and timetabling with passenger demand uncertainty, *Transportation Research Part B: Methodological*, **136**, 1–29, ISSN 0191-2615.

Cadarso, L., L. F. Escudero and A. Marín (2018) On strategic multistage operational two-stage stochastic 0–1 optimization for the rapid transit network design problem, *European Journal of Operational Research*, **271** (2) 577–593, ISSN 0377-2217.

Caimi, G., L. Kroon and C. Liebchen (2017) Models for railway timetable optimization: Applicability and applications in practice, *Journal of Rail Transport Planning Management*, **6** (4) 285–312, ISSN 2210-9706.

Feng, Z., C. Cao, A. Mostafizi, H. Wang and X. Chang (2023) Uncertain demand based integrated optimisation for train timetabling and coupling on the high-speed rail network, *International Journal of Production Research*, **61** (5) 1532–1555.

Fischetti, M. and M. Monaci (2009) Light robustness, *Robust and online large-scale optimization: Models and techniques for transportation systems*, 61–84.

Guihaire, V. and J.-K. Hao (2008) Transit network design and scheduling: A global review, *Transportation Research Part A: Policy and Practice*, **42** (10) 1251–1273, ISSN 0965-8564.

Han, B. and S. Ren (2020) Optimizing stop plan and tickets allocation for high-speed railway based on uncertainty theory, *Soft Computing*, **24**, 6467–6482.

Hao, L., J. Qin, X. Sarah Yang, W. Zhou and C. Xie (2023) Joint train line planning and timetabling of intercity high-speed rail with actual time-dependent demand, *International Journal of Transportation Science and Technology*, **12** (2) 534–548, ISSN 2046-0430.

Kall, P. and S. W. Wallace (1994) *Stochastic programming*, vol. 6, Springer.

Khan, M. B. and X. Zhou (2010) Stochastic optimization model and solution algorithm for robust double-track train-timetabling problem, *IEEE Transactions on Intelligent Transportation Systems*, **11** (1) 81–89.

Kroon, L., G. Maróti, M. R. Helmrich, M. Vromans and R. Dekker (2008) Stochastic improvement of cyclic railway timetables, *Transportation Research Part B: Methodological*, **42** (6) 553–570, ISSN 0191-2615.

Lusby, R. M., J. Larsen and S. Bull (2018) A survey on robustness in railway planning, *European Journal of Operational Research*, **266** (1) 1–15, ISSN 0377-2217.

Nie, B., L. Nie, H. Fu and X. Wu (2023) Weekly line planning to improve the match between rail supply and passenger demand, *Computer-Aided Civil and Infrastructure Engineering*, **38** (13) 1810–1833.

Pu, S. and S. Zhan (2021) Two-stage robust railway line-planning approach with passenger demand uncertainty, *Transportation Research Part E: Logistics and Transportation Review*, **152**, 102372, ISSN 1366-5545.

Qi, J., V. Cacchiani and L. Yang (2018) Robust train timetabling and stop planning with uncertain passenger demand, *Electronic Notes in Discrete Mathematics*, **69**, 213–220, ISSN 1571-0653. Joint EURO/ALIO International Conference 2018 on Applied Combinatorial Optimization (EURO/ALIO 2018).

Schiewe, A., A. Schöbel and L. Sieber (2023) Line planning for different demand periods, paper presented at the *Operations Research Forum*, vol. 4, 1–28.

Schöbel, A. (2012) Line planning in public transportation: models and methods, *OR spectrum*, **34** (3) 491–510.

Schöbel, A. (2014) Generalized light robustness and the trade-off between robustness and nominal quality, *Mathematical Methods of Operations Research*, **80** (2) 161–191.

Shakibayifar, M., E. Hassannayebi, H. Jafary and A. Sajedinejad (2017) Stochastic optimization of an urban rail timetable under time-dependent and uncertain demand, *Applied Stochastic Models in Business and Industry*, **33** (6) 640–661.

Wang, L., J. G. Jin, G. Sibul and Y. Wei (2023) Designing metro network expansion: Deterministic and robust optimization models, *Networks and Spatial Economics*, **23** (1) 317–347.

Yang, L., K. Li and Z. Gao (2009) Train timetable problem on a single-line railway with fuzzy passenger demand, *IEEE Transactions on Fuzzy Systems*, **17** (3) 617–629.

Yang, X., A. Chen, B. Ning and T. Tang (2017) Bi-objective programming approach for solving the metro timetable optimization problem with dwell time uncertainty, *Transportation Research Part E: Logistics and Transportation Review*, **97**, 22–37, ISSN 1366-5545.

Yao, B., P. Hu, X. Lu, J. Gao and M. Zhang (2014) Transit network design based on travel time reliability, *Transportation Research Part C: Emerging Technologies*, **43**, 233–248, ISSN 0968-090X. Special Issue on "Nature-Inspired Optimization Techniques in Transportation Planning and Operation".

Yin, J., T. Tang, L. Yang, Z. Gao and B. Ran (2016) Energy-efficient metro train rescheduling with uncertain time-variant passenger demands: An approximate dynamic programming approach, *Transportation Research Part B: Methodological*, **91**, 178–210, ISSN 0191-2615.

Zhou, W., Y. Huang, X. Hu, L. Deng and G. Xu (2023) A line planning approach based on time-varying demand for high-speed rail under the combined operation of periodic and aperiodic services, *Computers Industrial Engineering*, **185**, 109665, ISSN 0360-8352.

Şahin, G., A. Ahmadi Digehsara, R. Borndörfer and T. Schlechte (2020) Multi-period line planning with resource transfers, *Transportation Research Part C: Emerging Technologies*, **119**, 102726, ISSN 0968-090X.

A Overview of the Papers

Table 1: Overview of the papers. 'Det.' = Determining

Paper	Solving	Model	Uncertainty in	Modelling of the Stochasticity	Solution Method
An and Lo (2015)	Det. Lines & Flexible Service Det. Start & End of Lines Det. Frequencies Passenger routing	MILP Robustness (protection level)	demand	polyhedra uncertainty set (box uncertainty but limiting total travelers heading to same desination by a upper bound)	cutting constraint algorithm (Frank-Wolfe algorithm)
An and Lo (2016)	Passenger Routing Det. & Flexible Service Det. Start and End of Lines Det. Frequencies	2-stage model MILP	demand	for robustness: predetermined uncertainty set calculated from a continuous distribution for stoch: "appropriate" sampling method used for continuous distribution	Gradient Method Neighbourhood Search
Cacchiani *et al.* (2020)	Time Tabling Stop Plan Passenger Routing	MILP Light Robustness	protection against demand-increase	different for different models: protection level (additional passengers) for any two stations protection level for possible addition of passengers between two stations on a particular train	- (solved using CPLEX)
Cadarso *et al.* (2018)	Strategic phase: station construction Det. lines operational phase: passenger flows usage of lines	binary LP	strategic & operational cost, passenger demand, network disruption	scenario tree	3 algorithms presented based on fix-and-relax and lazy matheuristic algorithm with dynamic scenario (de)aggregaton
Feng *et al.* (2023)	Passenger Routing Adding additional-candidate trains Time Tabling Coupling	2-stage model MILP	passenger demand	s stochastic scenarios	Heuristic local search with branch-and bound
Han and Ren (2020)	Stop Plan Ticket allocation	MILP uncertainty theory	passenger demand	demand between station (and depending on seat grade) are uncertain variables with some ditribution $\Phi(c, i, j)$	Lagrangian relaxation heuristic algorithm
Hao *et al.* (2023)	Choosing trains Det. & end of train Det. routes of train Stop plan Passenger Routing Timetabling	MINLP	- (demand over time)	demand given for different planning steps	double-layer simulated annealing algorithm
Khan and Zhou (2010)	Time Tabling	2-stage model	travel-time, departure time	scenarios	heuristic sequential solution frame-work
Kroon *et al.* (2008)	Time Tabling	2-stage model	delays	R independent realizations of timetable subject to a priori slected stochastic primary disturbance (case study: exponential distribution based on data)	Sample Average Approximation Method (CPLEX)
Nie *et al.* (2023)	Passenger Routing Frequency Choosing Stopping pattern Choosing Lines	MILP	- (Multiperiod)	D days and t time intervals considered	genetic algorihtm

Pu and Zhan (2021)	Stop Plan Passenger Routing	Robust 2-stage model MILP	passenger demand	random daily passenger demand, in a given box set B	Lagrangian Relaxation
Qi *et al.* (2018)	Time Tabling Stop Plan Passenger Routing	Light Robustness ILP	passenger demand	protection level $\Delta_{i,j}$ introduced (how many additional number of passengers should be considered) for any two stations i, j	GAMS (CPLEX)
Şahin *et al.* (2020)	Choosing Lines Det. Capacity on lines Resource Transfer	ILP	- (Multiperiod on passenger demand)	demand for edges and periods given	Gurobi (IP solver)
Schiewe *et al.* (2023)	Choosing Lines Det. Frequency	ILP	- (Multiperiod on passenger demand)	n periods, upper and lower bounds for frequencies given (based on passenger demand and other input factors)	Gurobi (IP solver)
Shakibayifar *et al.* (2017)	Timetabling	2-stage model MILP and nonlinear formulation	stoch fluctuation of passenger arrival rates	scenarios: each realisation of the random variable has a probability p_ω (distribution estimated from historical data)	Sample average approximation method (sampling repeated a couple of times until optimality gap sufficiently low)
Wang *et al.* (2023)	Line Extension	2 models: Binary ILP Robust ILP	demand	uncertainty parameter takes value in a symmetric distribution set $[q - \delta_q, q + \delta_q]$	Section-based strategy, using CPLEX
Yang *et al.* (2009)	Time Tabling	fuzzy goal-Programming Model with mixed integer linear constraints	fuzzy passenger demand	fuzzy parameters	fuzzy simulation-based Branch and Bound
Feng *et al.* (2023)	Time Tabling Speed Profile	bi-objective expected value model (i.e. 2 expectations in obj)	dwell time	probability density function of stochastic dwell time given	genetic algorithm framework with epsilon-constraint method
Yao *et al.* (2014)	Det. Lines Passenger Routing	Mathematical Programming	travel time	for each link has K samples to consider take expectation + some risk attitude towards travel time	Tabu search
Yin *et al.* (2016)	Time Tabling	stochastic nonlinear-programming model	dynamic passenger demand	arrival of passengers at each station (with given destination) is a stochastic counting process denoted by a nonhomogeneous poisson process	approximate dynamic programming algorithm
Zhou *et al.* (2023)	Choosing Lines Stop Plan Passenger Routing Time Tabling	MINLP Multi-Period	- (demand is time-varying)	arrival rate at station for passengers with some destination at given time is given	simulated annealing algorithm (compared to particle swarm optimization algorithm in experimental studies)