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Challenges and Opportunities in Modelling Traffic Assignment

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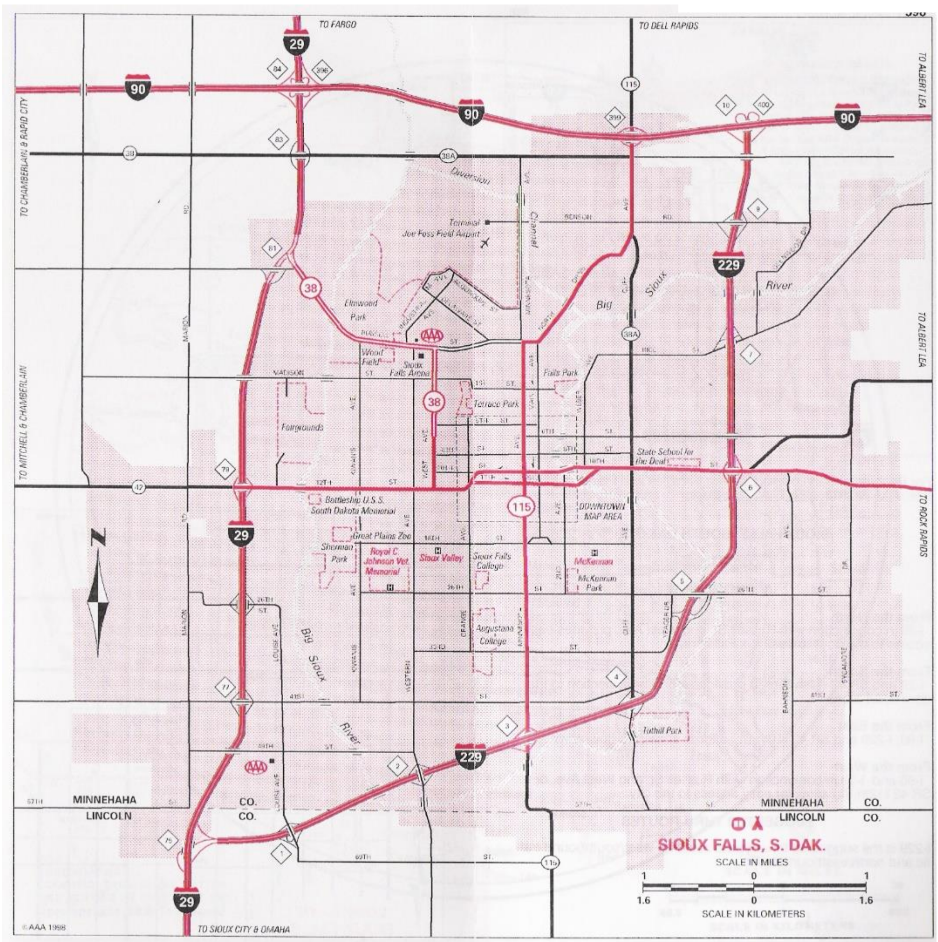
Summary and objectives

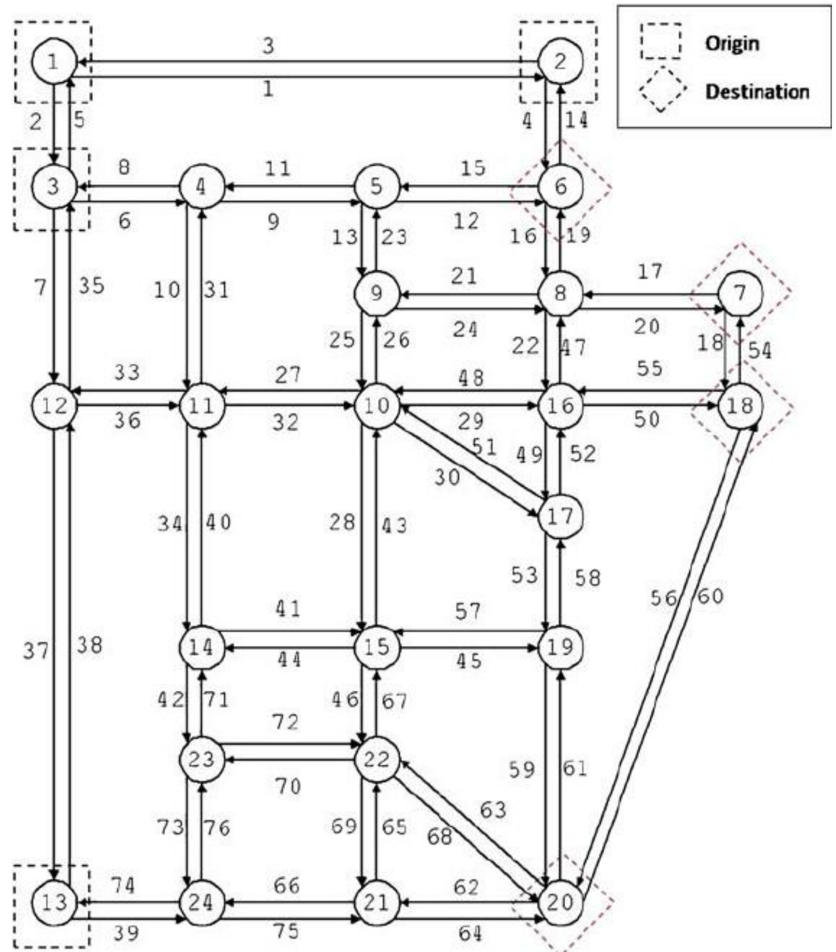
- Review of existing models for traffic assignment
  - focus on issues, caveats, hints, for model building
- Motivation
  - experts are too conservative
- Presentation of the Trust Contraction Algorithm
  - for solving traffic assignment as a fixed-point problem

Abstract
Traffic assignment is the main tool for transport engineers to simulate and compare design scenarios of intervention on a network taking into account user's reaction. This is a challenging mathematical problem, especially if we consider the dimensions and the complexity of real applications. Generations of researchers produced a quantity of models and algorithms to cover different use cases, many of which are today available in professional software. Yet, in practice, the large majority of the instances are still solved by adopting static deterministic assignment after a round of OD matrix estimation from traffic counts. This review of methods focuses on issues, caveats and hints, aiming at raising the awareness of modern challenges and opportunities, starting from facts that most modellers have probably experienced in their professional activities.

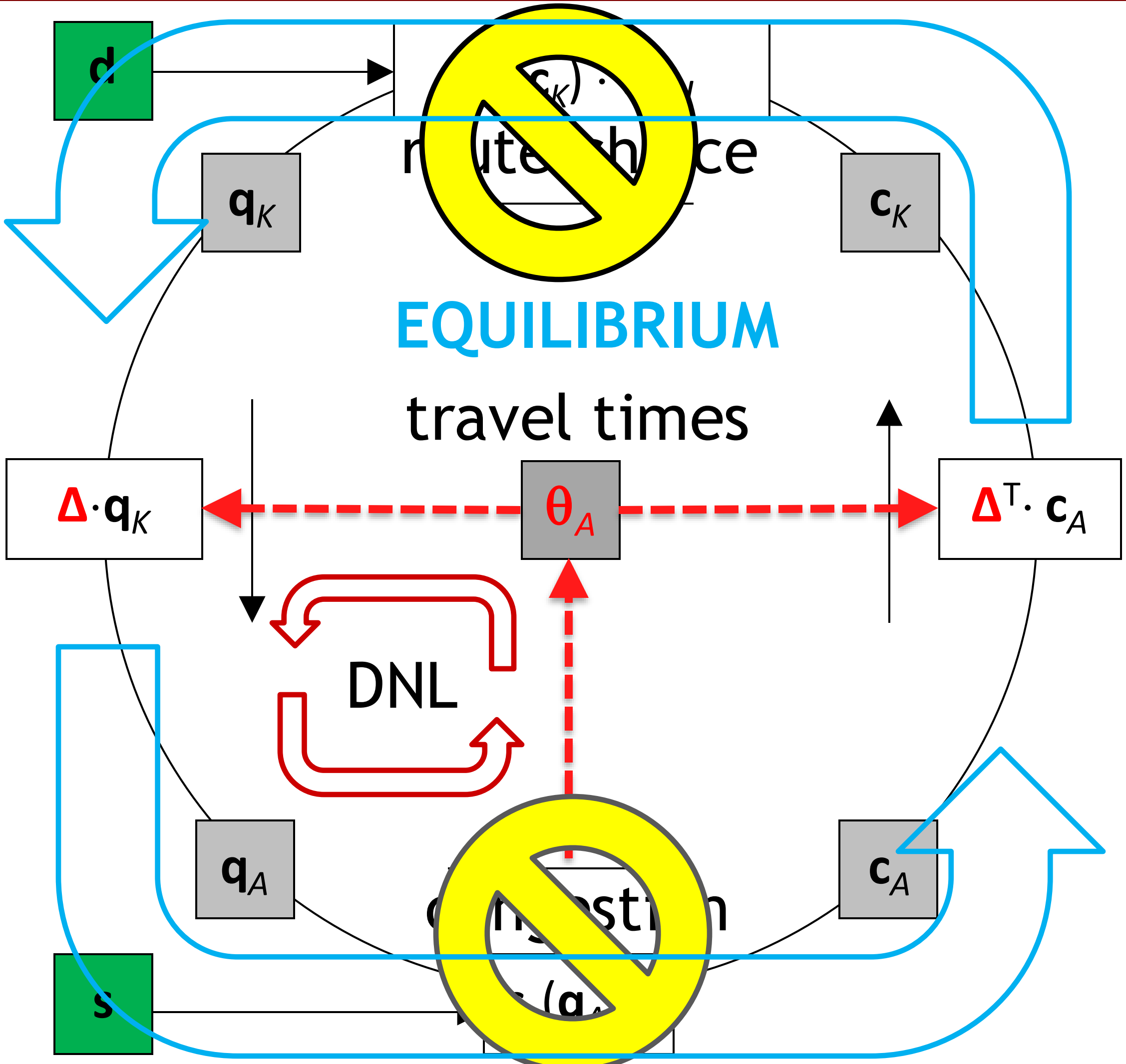
How travel demand uses the supply
i.e. the resulting flows and costs

Sioux Falls road network





General framework for assignment models



network topology

$i \in N$ nodes

$a \in A$ links

$z \in Z \subseteq N$ zones

$k \in K$ paths

variables - output

q flows

c costs

constants - input

d OD matrix demand

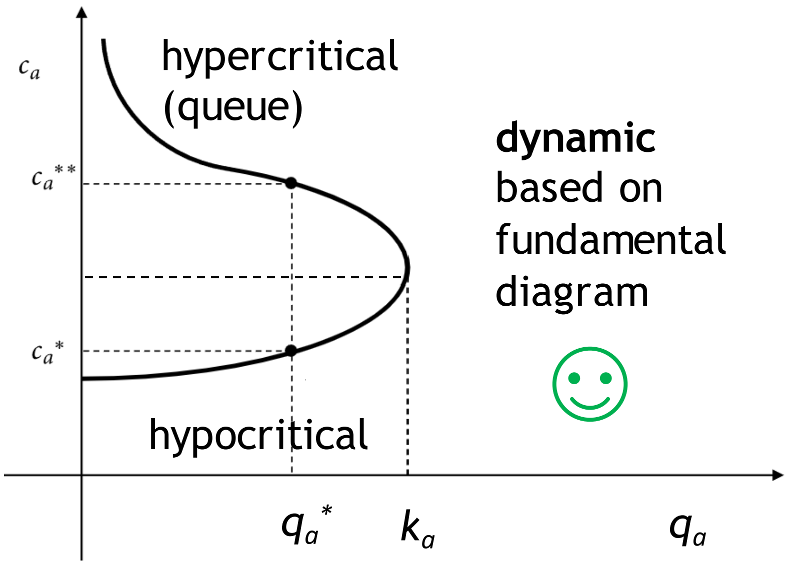
s capacity, free flow speed supply

Partial assignment models

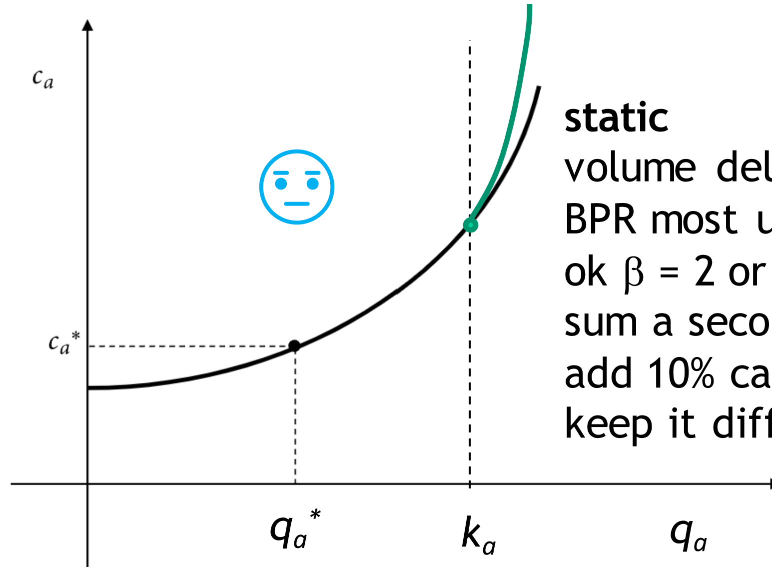
Congestion / Travel Times / Route Choice

- Uncongested networks
  - No congestion model (fixed travel times and costs)
- Network loading
  - Supply side congestion (only travel times)
  - No route choice model (fixed path flows)
  - Fixed Point Problem in a dynamic framework
- Equilibrium
  - Static → strategic planning – not realistic if queues matter
  - (within day) Dynamic → also for real time – evolution of queues
  - Congestion + Route Choice
  - Fixed Point Problem with Fork and Join for DNL

Separable arc cost functions
the role of capacity k and speed v



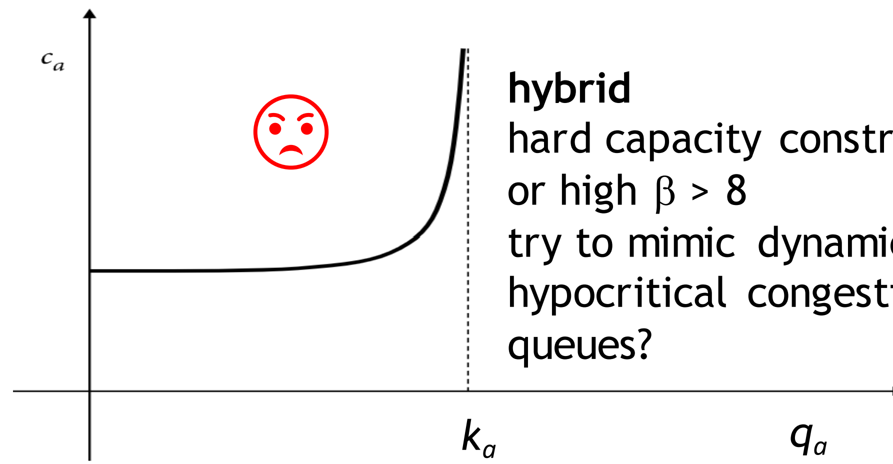
dynamic
based on
fundamental
diagram



static
volume delay
BPR most used
ok β = 2 or 4 or
sum a second β8 BPR
add 10% capacity
keep it differentiable

- If you can accept the limitations of static assignment, think it for the whole analysis period when queues will appear and vanish
- Hybrid is not realistic and can hinder model convergence

$$c_a = \frac{l_a}{v_a} \cdot \left[1 + \alpha_a \cdot \left(\frac{q_a}{k_a} \right)^{\beta_a} \right]$$



hybrid
hard capacity constraint
or high β > 8
try to mimic dynamic
hypocritical congestion?
queues?

Macro and Micro – pros and cons

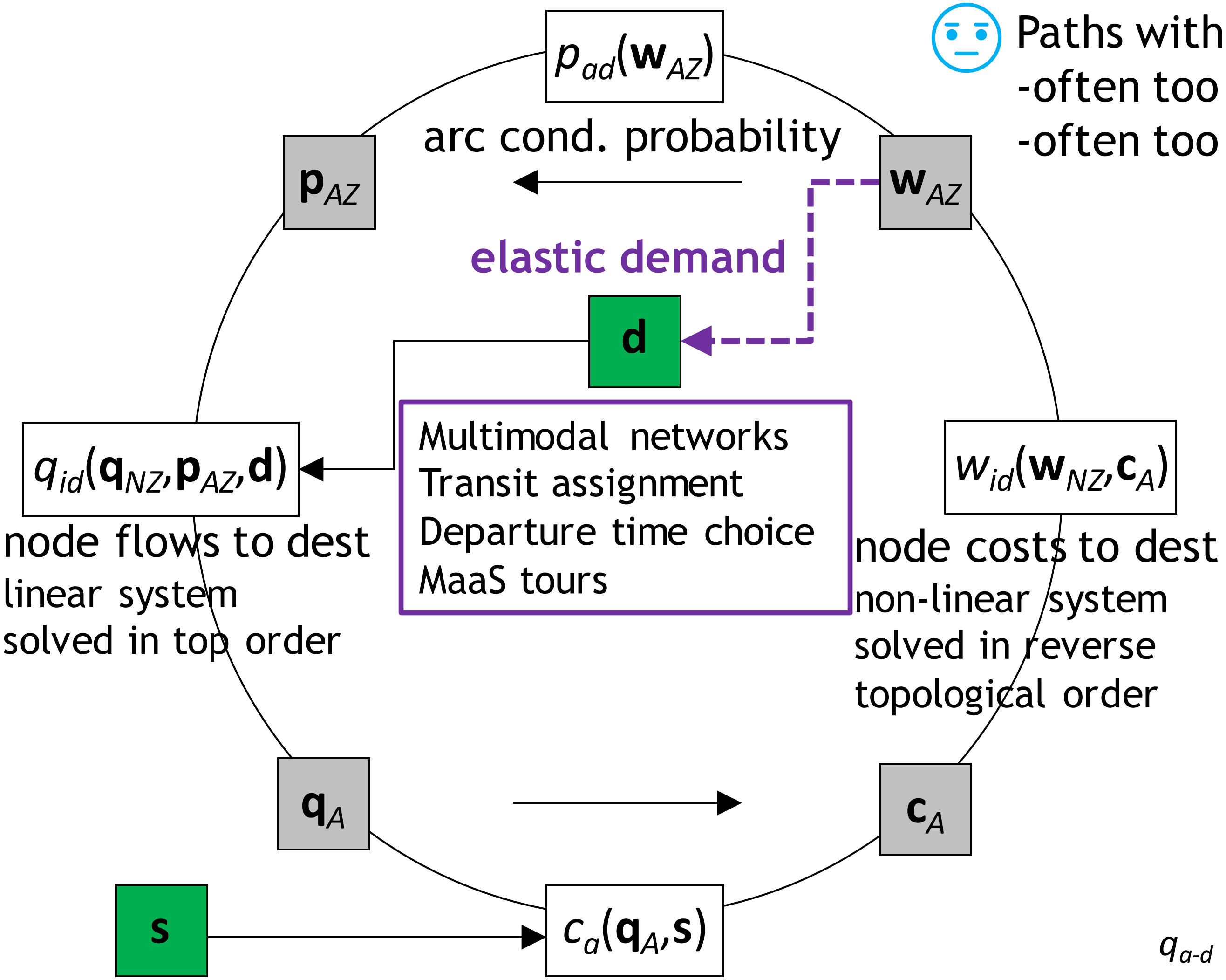
How to represent vehicles / users

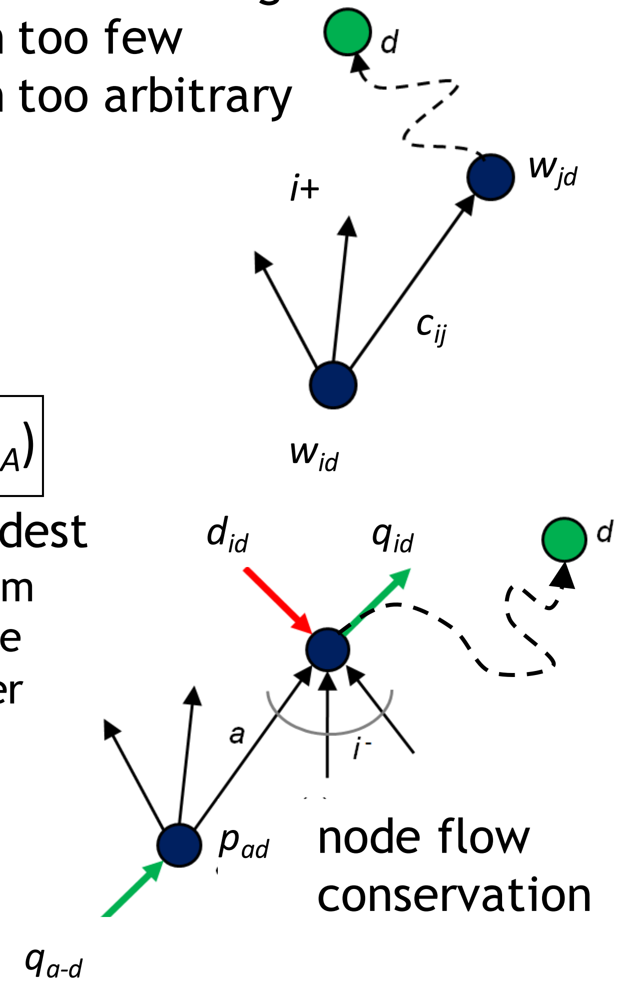
- Macroscopic models
  - Flows: vehicles are particles of a compressible fluid
  - Mathematically sound → Fast(er) and precise computation (convergence) → Robust scenario comparison
- Microscopic models
  - Detailed interactions among vehicles and with infrastructure (design of construction features)
  - Single travelling entities are reproduced (nice movie / trajectories)
  - Randomly extracted departures form given demand distribution
  - High computing times (suitable for corridors, no large networks)
  - Usually, no equilibrium – only network loading

Sequential route choice

implicit vs. explicit path enumeration

Paths with coloring
- often too few
- often too arbitrary





Route choice

why are we still using deterministic models?

- Deterministic
  - Traditionally, the way to go (Wardrop's conditions)
  - Unrealistic behavioural assumptions: rationality and perfect information (which one is worse?)
  - A complex model to formulate (many to one map)
  - Difficult algorithm convergence (despite gradient projection)
  - Only relatively few OD couples use more than one path
  - When coupled with OD estimation congestion is overestimated, because to shift flow on non-shortest paths you need congestion

Route choice
why stochastic models are not much used?

- Stochastic Logit
  - Sound, simple and in closed form, but no correlation
  - Can well mimic deterministic with small $\theta = 0.01$
  - Very smooth model and easy convergent algorithm for $\theta = 0.1$
  - Can be implemented as a deterministic model (perceived costs)
- Overlapping of paths → correlation of alternatives
  - Probit: requires to average several Montecarlo simulations
  - this is not practical for scenario comparison
  - C-Logit: it works well in practice, although in theory it is a trick
  - C-Logit: implicit enum. with tarjan lowest common ancestors
  - Correlation does not play a major role in practice

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Demand Aggregation
an estimation need

- With 1 thousand zones we have 1 million OD couples
- How can we pretend to estimate so many little demand variables?
- Intermediate models → aggregation, estimation, disaggregation
  - Clustering OD couples – explicit path enumeration
  - Clustering zones – implicit path enumeration
  - Target: 100 meaningful (urbanistic) macrozones in a medium city
- Disaggregation with connectors to all internal nodes
  - low urban speed (10-20km/h)
  - congestion to spread the flow → capacity = attraction / num. nodes
  - Demand can be loaded on all node origins and propagated to macro dest
- Computing times are reduced proportionally to the number of aggregated zones (relevant for real time applications)

Supply Aggregation
a forgotten modelling phase

- Getting the network from OSM or other commercial provider is common practice ☺
- The result is a very rich graph: say 100.000 links for a medium city
- But keeping local roads used only for access/ egress can lead to relevant distortions ☹
  - The algorithm may not simulate the reduced perception of users
  - Local roads get loaded as soon as major roads get congested
- Prune local and service roads → less intersections on major
- Keep high speed, high capacity and highly desired
  - Desired flows are measured by uncongested assignment

What do we put in the model
calibration of supply

- Start from digital copy (i.e. all OSM features)
- Calibration of BPR functions? Not so useful as approximation is native
  - Focus on free flow speed and capacity – set $\alpha = 1$ and $\beta = 2+4$
- Check capacity/lanes of critical corridors, often we see errors there
  - Use uncongested assignment to focus on highly desired roads
- Set the speed of local / parking roads to max 20km/h (not to 50km/h)
- Add delay of traffic lights (available in OSM)
  - For example add 30 sec to each link that ends closer than 30 m to a signal
- Connectors: add lots of them to avoid fictitious bottlenecks

Estimation of OD matrix
the illusion of traffic counts

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- Model calibration can be thought as a system of non-linear equations
  - which can be linearized at a given point
- In regression we expect many points to determine a few parameters
  - rows >> columns
- Here we are in the opposite situation: some hundred link flow measurements to determine some thousands demand flows
  - If we add measured costs (FCD speed profiles) → still columns >> rows
- The classical problem is not well posed: od flows will be set to almost match traffic counts, but many possible demand patterns exist that can achieve a similar result
- The number of variables to estimate should be comparable with that of measurements, i.e. we should use counts to estimate the parameters of a model that generates the OD matrix

Estimation of Demand
reconsider traditional 2/3 steps with new data

- The existing OD matrix is most probably out of date
  - Nobody dares questioning the work of the previous distinguished colleague
  - But we know that adjustment based on traffic counts is just a final polish
- For the most difficult challenge → invest resources and use models
- Generation and Attraction: robust models → pop x rate
  - Residents demography from census data (classical but effective)
  - Pois activities of Google (very promising) ☺
- Distribution: (new) sources of data → caveat sampling issues 😐
  - Mobile phone data: OD matrices
  - GPS data of floating cars: speed profiles ☹, trajectories (also route choice) ☺
- Mode choice: mobility management survey → travel journal
- Estimation based on traffic counts of model aggregated parameters

Traffic Assignment
as fixed point problem

- **Fixed-point problems** occur in many fields to describe equilibrium
- They can be considered a particular case of squared systems of nonlinear equations, and are intimately connected with **unconstrained optimization**
- Find $\mathbf{x} \in X$ such that: $f(\mathbf{x}) = \mathbf{x}$, where
  - the **map** f is a continuous vector function (defined everywhere on R_n)
  - the **feasible set** X is non-empty, compact and convex
  - the codomain is contained in the domain: $f(R_n) \subseteq X$
- According to Brouwer theorem these hypotheses ensure the existence of a sol.
- The focus of the fixed-point problem is hence on the map, rather than on the feasible set, since the application of the function implicitly satisfies the constraints
- Equivalent to solve a **squared system of nonlinear equations**: $y(\mathbf{x}) = f(\mathbf{x}) - \mathbf{x} = \mathbf{0}$
- Consider the **sum of the residual squares**: $r(\mathbf{x}) = || y(\mathbf{x}) ||^2$ → objective function
  - null only at fixed points and positive elsewhere: can be used to measure the distance to a sol. at $\mathbf{x}$
- Path based static equilibrium: $\mathbf{x}$ is $\mathbf{q}_K$ and $f(\mathbf{x})$ is $p_K(\Delta^T \cdot c_A(\Delta \cdot \mathbf{q}_K)) \times \mathbf{d}$

MSA and its variants

- **Method of Repeted Approximations** : $\mathbf{x}_{i+1} \leftarrow f(\mathbf{x}_i)$ equilibrium is not a contraction
- The **Method of Successive Averages** is widely appreciated for its generality
- But for many problems the MSA is known to be too slow in convergence
- This low practical precision may prevent the usage of more appropriate assignment models, in particular for scenario comparison
- MSA: $\mathbf{x}_{i+1} \leftarrow \mathbf{x}_i + \alpha_i \cdot (f(\mathbf{x}_i) - \mathbf{x}_i)$ Blum: $\alpha_i \leftarrow 1 / (1 + \gamma \cdot (i\text{-}1))$
  - The step $\alpha_i \in (0,1]$ decreases inversely to the number of iterations i
  - Since X is convex, all the points of the segment between **x** and f(**x**) belong to the feasible set
  - Provided that the solution is unique, the method can be proved to converge under the following regularity conditions for the step size (Blum, 1954): $\sum_{i=1}^{\infty} \alpha_i = \infty \quad , \quad \sum_{i=1}^{\infty} \alpha_i^2 < \infty$
- For α_i = 1/i we have the original MSA of Robbins and Monro (1951), where the current iterate is the average of all previous i function evaluations, which makes the method robust but inevitably slow
- Polyak and Juditsky (1992) proposed a step-size equal to $\alpha_i = 1/i^{2/3}$ to gain convergence speed, because new function evaluations are weighted more than old

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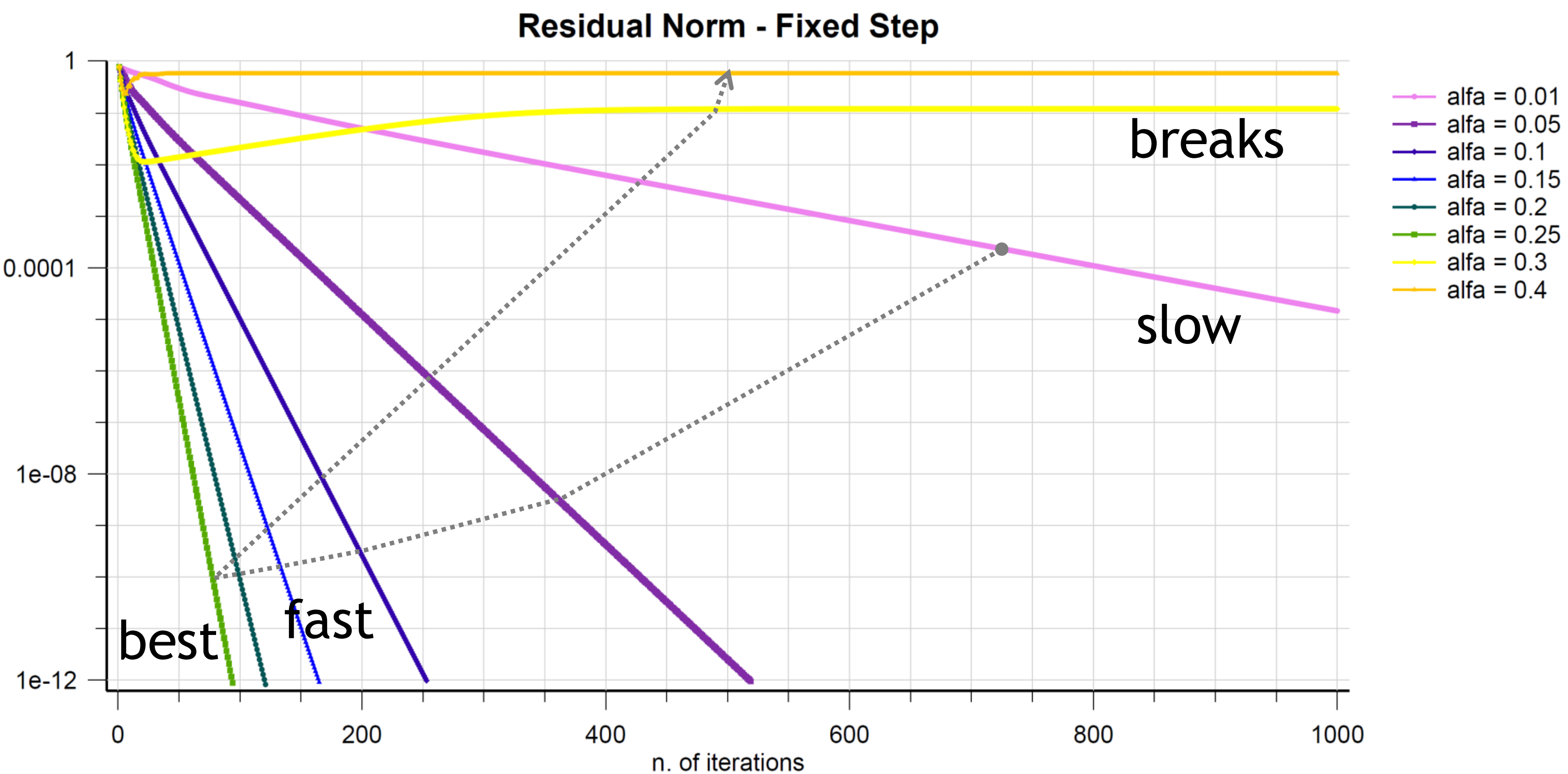
Trust Contraction Algorithm

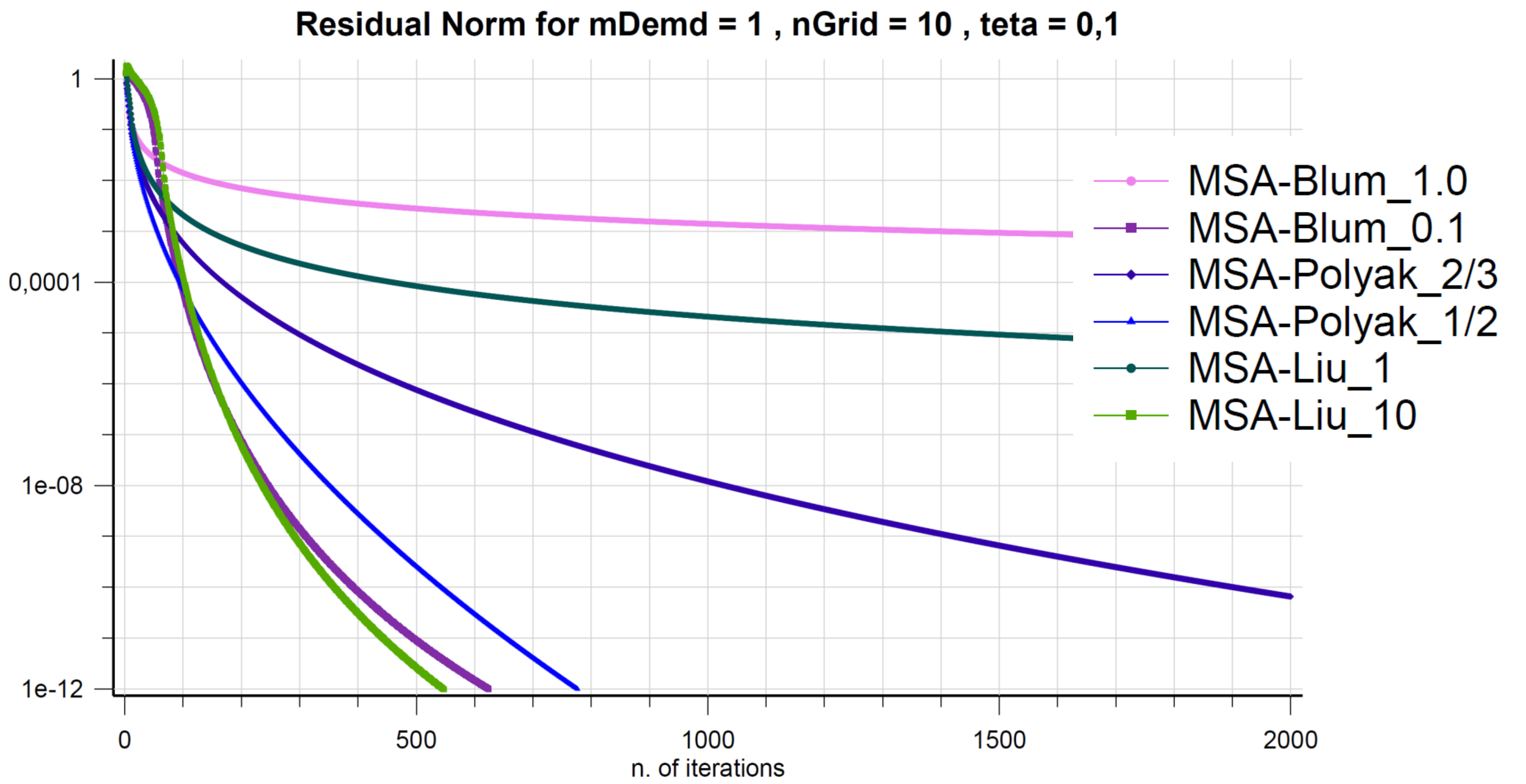
- In the following, a simple alternative to MSA is presented, which proves to converge much faster for most traffic assignment models
- Assume that $y(\mathbf{x}) = f(\mathbf{x}) - \mathbf{x}$ is a **descent direction** wrt to the sum of residual squares
- Two **damping** strategies are explored to determine the step α from $\mathbf{x}$ to $f(\mathbf{x})$
- 1. Backtracking line search of Armijo
  - The step α is progressively reduced by 2 until the candidate solution $\hat{\mathbf{x}}$ sufficiently improves the current iterate $\mathbf{x}$, i.e.: $r(\hat{\mathbf{x}}) < r(\mathbf{x})$. Only at this point, the current iterate is updated, as well as the search direction, while α is increased by 1.1 in the hope that with a larger initial step, the next backtracking search may reach lowest residuals, instead of resetting the step to one
- 2. Adaptive method, which seeks convergence for a fixed, but unknown, step size
  - The step α is reduced by 2 after bad iterations and increased by 1.1 after good iterations
- The name Trust Contraction of the proposed algorithm is thus explained
  - given that the "contraction" direction is descent, the step shall belong to a "trust" region, which implies a suitable dumping of α from the pure contraction value of 1
- The (negative) **semi-definiteness** of the fixed-point map **Jacobian** is a sufficient condition for the uniqueness as well as for the convergence

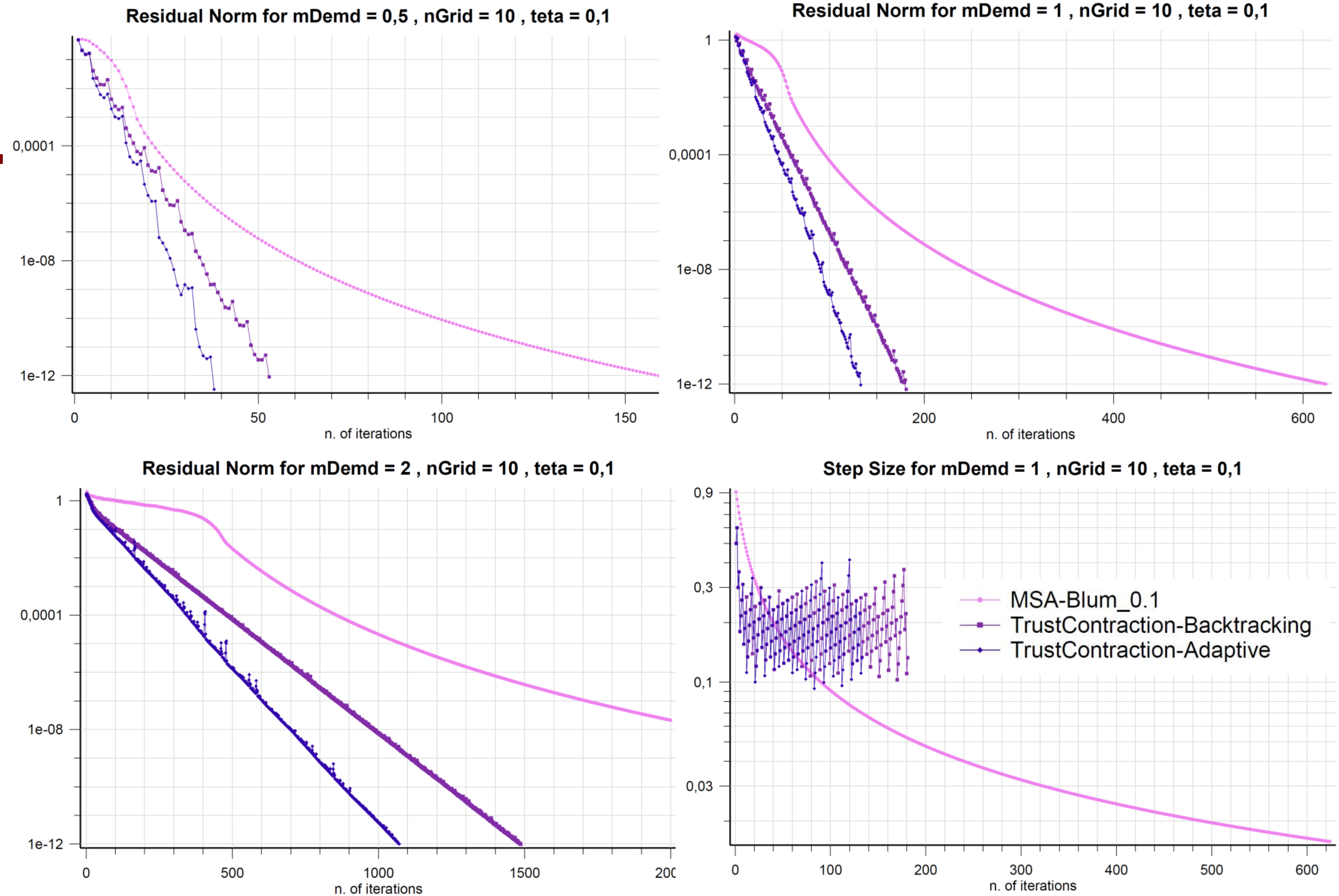
Fixed step size

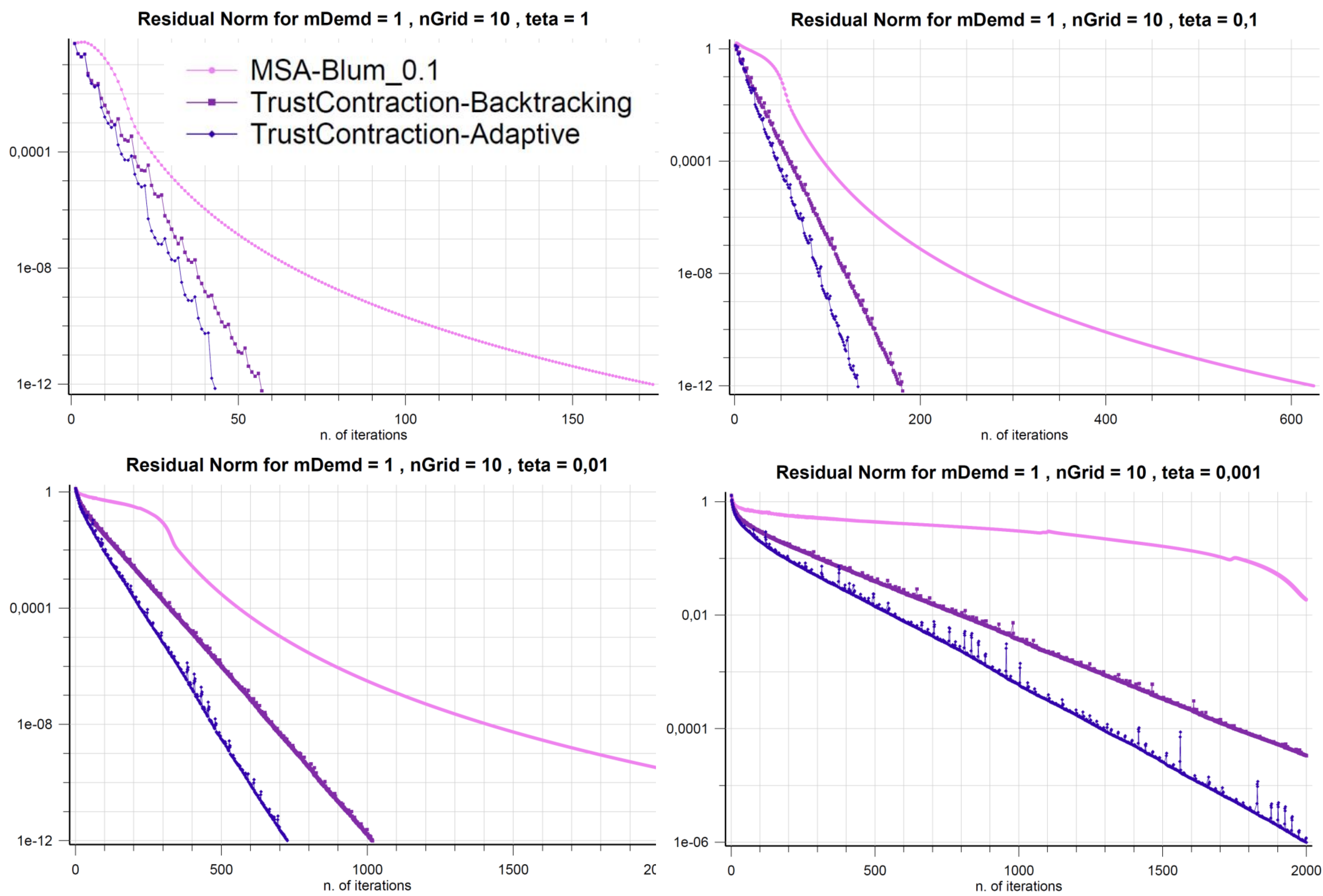
Sioux Falls – Logit route choice



Best MSA algorithm
Random grid network – Logit route choice

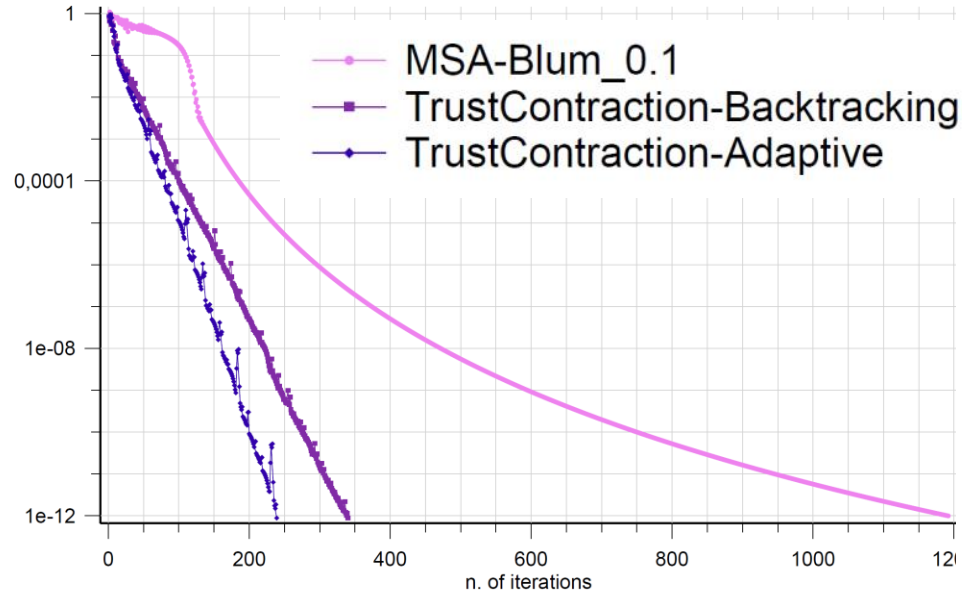



Residual Norm for mDemd = 0,5 , nGrid = 10 , teta = 0,1
0,0001
1e-08
1e-12
0
50
100
150
n. of iterations
Residual Norm for mDemd = 1 , nGrid = 10 , teta = 0,1
1
0,0001
1e-08
1e-12
0
200
400
600
n. of iterations
Residual Norm for mDemd = 2 , nGrid = 10 , teta = 0,1
1
0,0001
1e-08
1e-12
0
500
1000
1500
n. of iterations
Step Size for mDemd = 1 , nGrid = 10 , teta = 0,1
0,9
0,3
0,1
0,03
0
100
200
300
400
500
600
n. of iterations
MSA-Blum_0.1
TrustContraction-Backtracking
TrustContraction-Adaptive

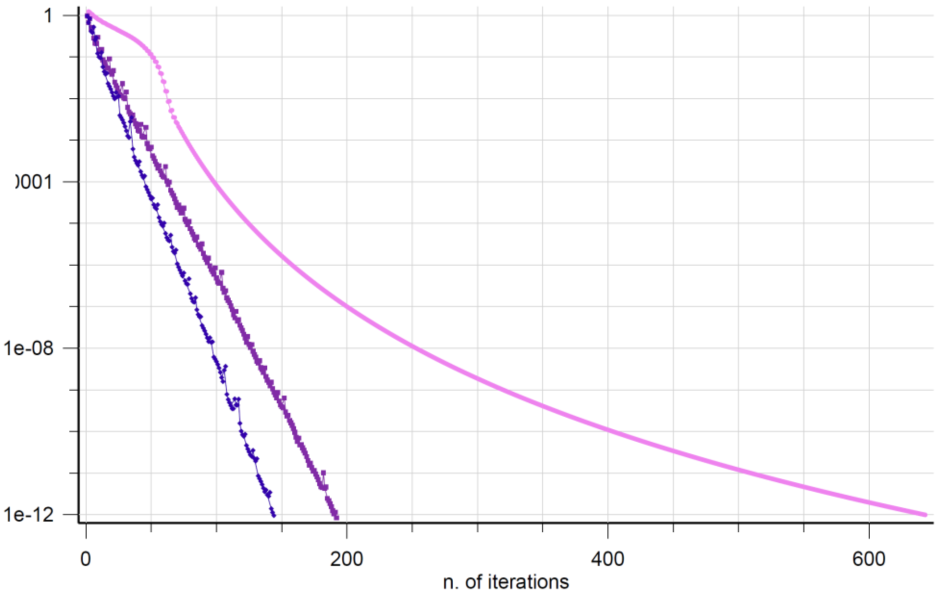


Residual Norm for mDemd = 1 , nGrid = 10 , teta = 1
MSA-Blum_0.1
TrustContraction-Backtracking
TrustContraction-Adaptive
0,0001
1e-08
1e-12
0
50
100
150
n. of iterations
Residual Norm for mDemd = 1 , nGrid = 10 , teta = 0,1
1
0,0001
1e-08
1e-12
0
200
400
600
n. of iterations
Residual Norm for mDemd = 1 , nGrid = 10 , teta = 0,01
1
0,0001
1e-08
1e-12
0
500
1000
1500
n. of iterations
Residual Norm for mDemd = 1 , nGrid = 10 , teta = 0,001
1
0,01
0,0001
1e-06
0
500
1000
1500
2000
n. of iterations

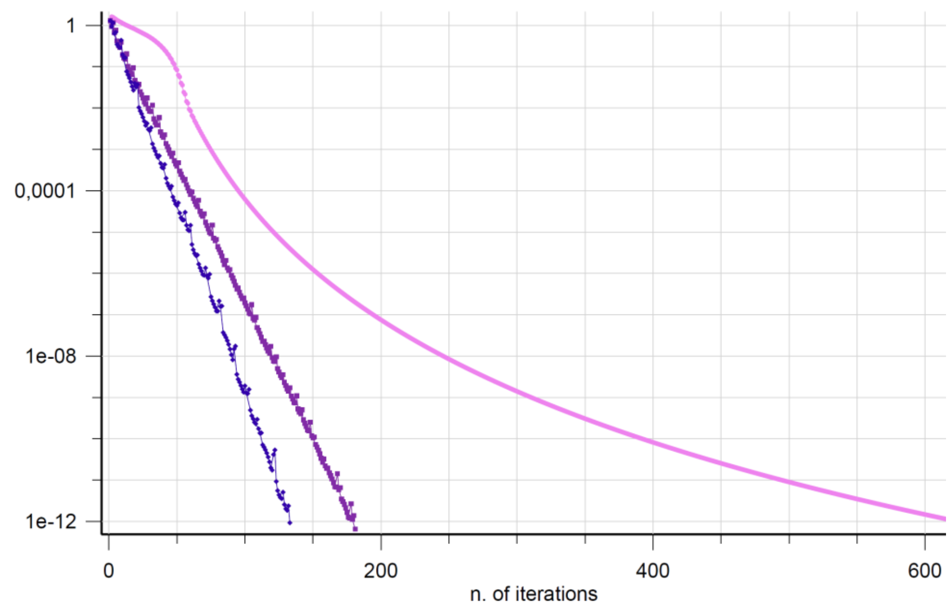
Residual Norm for mDemd = 1 , nGrid = 3 , teta = 0,1



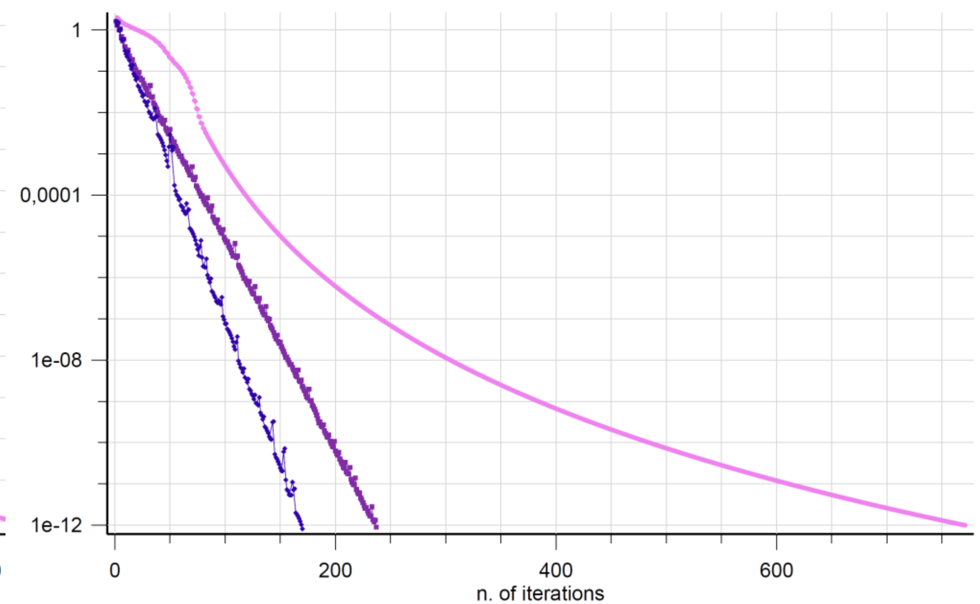
Residual Norm for mDemd = 1 , nGrid = 5 , teta = 0,1



Residual Norm for mDemd = 1 , nGrid = 10 , teta = 0,1

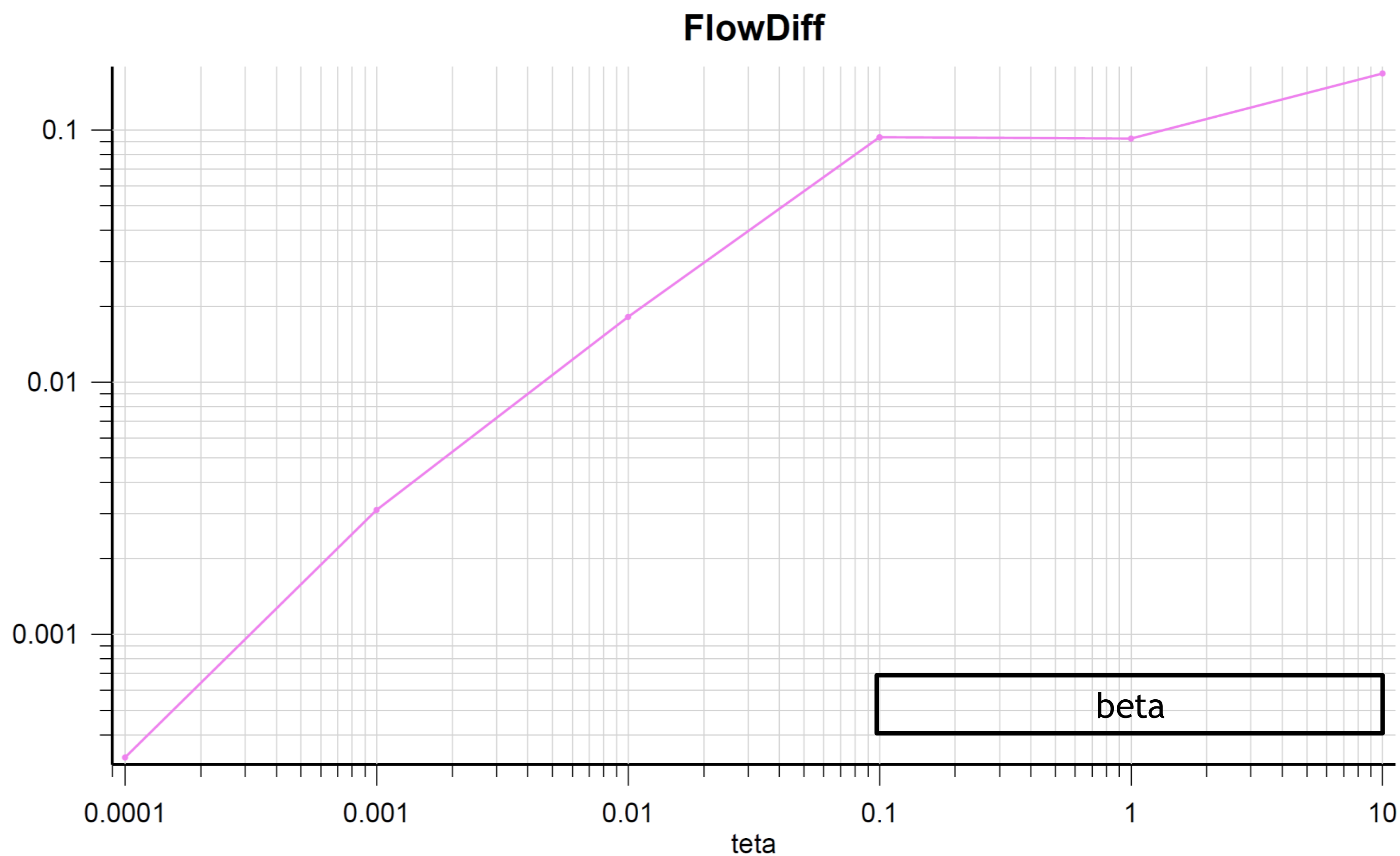


Residual Norm for mDemd = 1 , nGrid = 15 , teta = 0,1



The relevance of stochastic models

Sioux Falls



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Conclusions

- There are many models for traffic assignment
- But the community still uses mainly
  - deterministic static equilibrium
  - microscopic network loading
- Trust Contraction is a new general algorithm for fixed point problems
  - Very simple to implement (a couple of additional code lines wrt MSA)
  - Much faster and robust than MSA for traffic assignment: linear vs sublinear
- Convergence speed α* is inversely related to the Lipschitz continuity constant λ, which is increased by $\left\| f(\mathbf{x}_1) - f(\mathbf{x}_2) \right\| \leq \lambda_f \cdot \left\| \mathbf{x}_1 - \mathbf{x}_2 \right\| \quad , \forall \mathbf{x}_1, \mathbf{x}_2 \in X$
  - Sharp route choice (like deterministic)
  - Sharp cost changes (high congestion)
- Future work
  - The study of nonlinear system of equations paves the way to faster calibration of assignment models (automatic differentiation)