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Abstract

An alternative approach for real-time network-wide control that has recently gained a lot of interest is the perimeter flow control. The basic concept of such an approach is to partition heterogeneous cities into a small number of homogeneous regions (zones) and apply perimeter control to the inter-regional flows along the boundaries between regions. The inter-transferring flows are controlled at the intersections located at the borders between regions, so as to distribute the congestion in an optimal way and maximize the total throughput of the system. In the current work we focus on the same problem described above, studying three aspects that are not covered in the literature: (a) the uncertainties in some model parameters that are not measurable in real life implementations and can affect the performance of the controller (advanced estimation schemes can be developed to estimate these parameters in real-time implementations), (b) integration of external demand information that has been considered system disturbance in the derivation of feedback control in previous works, and (c) mathematical formulation of the original nonlinear problem in a convex optimization form, so that optimal control can be applied in a (rolling horizon) model predictive concept. This work presents the mathematical analysis of the optimal control problem, as well as the approximations and simplifications that are assumed in order to derive the convex formulation. Preliminary simulation results for the case of a macroscopic environment (plant)are presented, in order to demonstrate the efficiency and performance of the proposed approach.

Keywords

Macroscopic fundamental diagram; model predictive control; convex optimization; urban perimeter control; parameter estimation.

1 Introduction

Traffic congestion is a major problem of urban environments and modern metropolitan areas. Most cities around the world become persistently denser and wider over the last decades and the problem of urban traffic management is steadily gaining momentum due to its economic, social and environmental impacts. Many efforts have been carried out to optimize the signal settings during the peak hours, during which networks face serious congestion problems and the performance of the infrastructure degrades significantly. The state-of-practice strategies fail to deal efficiently with oversaturated conditions (i.e. queue spillbacks and partial gridlocks), as they are either designed by use of simplified models that do not accurately replicate the traffic flow phenomena (e.g. propagation of congestion), or they are based on application-specific heuristics.

An alternative approach for real-time network-wide control that has recently gained a lot of interest is the perimeter flow control (or gating). The basic concept of such an approach is to partition heterogeneous cities into a small number of homogeneous regions (zones) and apply perimeter control to the inter-regional flows along the boundaries between regions. The inter-transferring flows are controlled at the intersections located at the borders between regions, so as to distribute the congestion in an optimal way and minimize the total delay of the system (an alternative objective could be to maximize the total throughput). This can be viewed as a high-level regional control scheme and may be combined with other strategies (e.g. local, distributed or coordinated controllers) in a hierarchical control framework (as a matter of fact this topic has gained a lot of attraction in the research community lately). For a recent review on this research direction the reader is referred to Daganzo (2007), Keyvan-Ekbatani *et al.* (2012), Geroliminis *et al.* (2013), Aboudolas and Geroliminis (2013), Ramezani *et al.* (2015), Kouvelas *et al.* (2015, 2016).

In the current work we focus on the same problem described above and we study a convex formulation of an optimal control problem. We investigate three aspects of the problem that have not been covered in the literature: (a) the uncertainties in some model parameters that are not measurable in real life implementations and can affect the performance of the controller, (b) integration of external demand information that has been considered system disturbance in the derivation of feedback control in previous works, and (c) mathematical formulation of the original nonlinear problem in a convex optimization form. The original model for the dynamics of the multi-region process (plant) is highly nonlinear and the modelling tool that is utilized is the Macroscopic Fundamental Diagram (MFD). MFD provides a concave, low-scatter relationship between network vehicle accumulations [*veh*] or density [*veh/km*] and network circulating flow [*veh/h*] or production [*veh·km*] for every region of the system. The proposed methodology includes the real-time estimation of some model parameters from measurements and the inference of a simple prediction model from real data. The problem is solved in a rolling optimization time horizon, by deriving a model predictive control (MPC) framework and the control decisions are applied to the nonlinear plant for evaluation. Different objective functions are investigated and the efficiency of the control decisions is compared to the "benchmark" case, where the nonlinear MPC problem is solved using advanced nonlinear numerical solvers. Simulation results for the case of a macroscopic model (plant) are presented. Note that this "benchmark" approach is more challenging for application in real life due to computational requirements, and most importantly, the lack of detailed data. In the real world data availability constraints the methodologies that can be applied, and as a consequence the real-time applicability of the "benchmark" approach is considered cumbersome.

2 Methodology

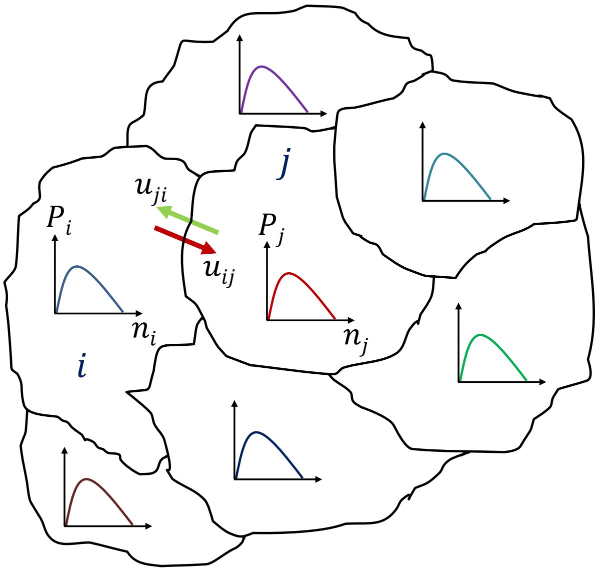
Consider an urban network partitioned in N homogeneous regions with well-defined MFDs (Figure 1). The index $i \in \mathcal{N} = \{1, 2, \ldots, N\}$ denotes the region of the system, $n_i(t)$ the total accumulation (number of vehicles) in region i and $n_{ij}(t)$ the number of vehicles in region i with final destination region $j \in \mathcal{N}$, at a given time t. Let $\mathcal{N}_i$ be the set of all regions that are directly reachable from the borders of region i, i.e. adjacent regions to region i. The discrete time MFD dynamics of the N-region system can be described by the following first order difference equations

$$n_{ii}(k_\mathrm{m}+1) = n_{ii}(k_\mathrm{m}) + T_\mathrm{m}\left(q_{ii}(k_\mathrm{m}) - M_{ii}(k_\mathrm{m}) - \sum_{h\in\mathcal{N}_i} M_{ii}^h(k_\mathrm{m}) + \sum_{h\in\mathcal{N}_i} M_{hi}^i(k_\mathrm{m})\right), \quad i \in \mathcal{N} \tag{1}$$

$$n_{ij}(k_\mathrm{m}+1) = n_{ij}(k_\mathrm{m}) + T_\mathrm{m}\left(q_{ij}(k_\mathrm{m}) - \sum_{h\in\mathcal{N}_i} M_{ij}^h(k_\mathrm{m}) + \sum_{h\in\mathcal{N}_i} M_{hj}^i(k_\mathrm{m})\right), \quad i, j \in \mathcal{N}, i \neq j \tag{2}$$

where $k_\mathrm{m} = 0, 1, \ldots, K_\mathrm{m} - 1$ is the model discrete time index, T_m [*sec*] the sample time period of the model (i.e. time $t = k_\mathrm{m}T$) and $n_i(k_\mathrm{m}) = \sum_{j\in\mathcal{N}} n_{ij}(k_\mathrm{m})$. The exogenous variables $q_{ij}(k_\mathrm{m})$ [*veh/sec*] denote the (uncontrollable) traffic flow demand that is generated in region i at time step k_m with final destination in region j (i.e. $q_{ii}(k_\mathrm{m})$ is the demand generated in region i that has final destination in region i). The variables $M_{ij}^h(k_\mathrm{m})$ [*veh/sec*] denote the transfer flows from region i to region h that have final destination region j, while $M_{ii}(k_\mathrm{m})$ [*veh/sec*] is the internal trip completion rate of region i (without going through another region).

Figure 1: The network of a city partitioned in a multi-region MFD system.



We assume that for each region i there exists a production MFD between accumulation $n_i(k_\mathrm{m})$ and total production $P_i(n_i(k_\mathrm{m}))$ [*veh·m/sec*], which describes the performance of the system in an aggregated way. This MFD can be easily estimated using measurements from loop detectors and/or GPS trajectories. Transfer flows $M_{ij}^h(k_\mathrm{m})$ and internal trip completion rates $M_{ii}(k_\mathrm{m})$ are calculated according to the corresponding production MFD of the region and proportionally to the accumulations $n_{ij}(k_\mathrm{m})$ as follows

$$M_{ii}(k_\mathrm{m}) = \theta_{ii}(k_\mathrm{m})\frac{n_{ii}(k_\mathrm{m})}{n_i(k_\mathrm{m})}\frac{P_i(n_i(k_\mathrm{m}))}{L_i}, \quad i \in \mathcal{N} \tag{3}$$

$$M_{ij}^h(k_\mathrm{m}) = \min\left\{C_{ij}^h(n_h(k_\mathrm{m})), u_{ih}(k_\mathrm{c})\theta_{ij}^h(k_\mathrm{m})\frac{n_{ij}(k_\mathrm{m})}{n_i(k_\mathrm{m})}\frac{P_i(n_i(k_\mathrm{m}))}{L_i}\right\}, \quad i, j \in \mathcal{N}, h \in \mathcal{N}_i \tag{4}$$

where $k_\mathrm{c} = 0, 1, \ldots, K_\mathrm{c}$ is the control discrete time index, L_i is the average trip length for region i, which is assumed to be independent of time and destination, internal (inside region i) or external (to some other region j). Note that the model discrete time index k_m must always be a multiple of the control discrete time index k_c, i.e. $k_\mathrm{m} = \beta k_\mathrm{c}$ must always hold for some integer β. This implies that the control cycle is always a multiple of the model sample time in order to avoid numerical issues. The parameters $\theta_{ii}(k_\mathrm{m})$, $\theta_{ij}^h(k_\mathrm{m})$ reflect the route choice and are assumed to be exogenous (i.e. can be constant or time varying and they are provided by another methodology). The transfer flows $M_{ij}^h(k_\mathrm{m})$ are the minimum between the sending flow from region i (which only depends on the accumulations of the region), and the receiving capacity $C_{ij}^h(n_h(k_\mathrm{m}))$ [*veh/sec*] of region h. This flow capacity is a piecewise function of the accumulation $n_h(k_\mathrm{m})$ (usually modelled with two pieces, one constant value and a decreasing curve) and is introduced to prevent overflow phenomena within the regions, i.e. each region i has a maximum accumulation $n_{i,\max}$

$$0 \leq n_i(k_\mathrm{m}) \leq n_{i,\max}, \forall i \in \mathcal{N}. \tag{5}$$

If $n_i(k_\mathrm{m}) = n_{i,\max}$ the region reaches gridlock and all the inflows along the periphery are restricted. Finally, the control variables $u_{ih}(k_\mathrm{c}), \forall i \in \mathcal{N}, h \in \mathcal{N}_i$ denote the fraction of the flow that is allowed to transfer from region i to region h at time k_c. Physical constraints are applied to the values of the control variables as follows

$$0 \leq u_{ih}(k_\mathrm{c}) \leq 1, \quad \forall i \in \mathcal{N}, h \in \mathcal{N}_i. \tag{6}$$

Equations (1)–(4) are a discretized version of equations presented in Yildirimoglu *et al.* (2015) and represent the traffic dynamics of an N-region urban network considering the heterogeneity effect and integrating an aggregated routing model. Note, that these equations allow the drivers to choose any arbitrary sequence of regions as their route and their path can cross region boundaries multiple times.

2.1 Nonlinear model predictive control (NMPC)

The MFD dynamics described in the previous section derive a nonlinear model that has been used in other works (Geroliminis *et al.,* 2013, Ramezani *et al.,* 2015) to apply nonlinear model predictive control (NMPC). Here, we solve the same problem again just to have it as a benchmark to evaluate the results that we get from the convex formulation presented later. In order to have a well defined problem and without loss of generality – since this is a nonlinear MPC problem – the following assumptions are made for formulating the problem:

- the quantities $q_{ii}(k_\mathrm{m})$, $q_{ij}(k_\mathrm{m})$ and $\theta_{ii}(k_\mathrm{m})$, $\theta_{ij}^{h}(k_\mathrm{m})$ are considered exogenous variables that can be measured or given by another algorithm beforehand,
- as in many similar works, the capacity constraint $C_{ij}^{h}(n_h(k_\mathrm{m}))$ in (4) is dropped since from a control viewpoint it is not necessary; the control actions will not allow the system to operate to states close to gridlock and this constraint is never activated inside the NMPC.

Given these two reasonable assumptions the nonlinear optimal control problem for a horizon of K_m model steps is defined as follows

$$\underset{n_{ii}(k_\mathrm{m}),n_{ij}(k_\mathrm{m}),u_{ih}(k_\mathrm{m})}{\text{minimize}} \quad \Theta(\mathbf{n}(K_\mathrm{c})) + \sum_{k=0}^{K_\mathrm{m}-1} \Phi(\mathbf{n}(k)) \tag{7}$$

subject to

equations (1), (2), (3) (5), (6)

$$M_{ij}^h(k_\mathrm{m}) = u_{ih}(k_\mathrm{m})\theta_{ij}^h(k_\mathrm{m})\frac{n_{ij}(k_\mathrm{m})}{n_i(k_\mathrm{m})}\frac{P_i(n_i(k_\mathrm{m}))}{L_i}, \quad i,j \in \mathcal{N}, h \in \mathcal{N}_i \tag{8}$$

$$n_i(k_\mathrm{m}) = \sum_{j \in \mathcal{N}} n_{ij}(k_\mathrm{m}), \quad i \in \mathcal{N} \tag{9}$$

$$k_\mathrm{m} = 0, 1, \ldots, K_\mathrm{m} - 1 \tag{10}$$

$$k_\mathrm{c} = 0, 1, \ldots, K_\mathrm{c} - 1 \tag{11}$$

where $\mathbf{n} = \mathrm{vec}(n_i)$ and $\Theta(\cdot)$, $\Phi(\cdot)$ derive the objective function (e.g. total delay, throughput, etc.). This problem can be solved by using advanced nonlinear optimization toolboxes (e.g. `ipopt`[1]) and in our case serves as a benchmark for the results reported later. Note that if we want to compare the results with the convex approach described later, the objective function should be the same in both cases in order to have a fair comparison and as a results this limits our choices only to convex functions.

2.2 Convexifying the problem

In the current work we derive a convex approximation of the model described earlier and we formulate a convex MPC problem that can be utilized for real-time control purposes. In order to convexify the dynamic equations we assume the following simplifications and approximations:[2]

[1] `http://www.i2c2.aut.ac.nz/Wiki/OPTI/index.php/Solvers/IPOPT`
[2] Note that for simplicity the time index is omitted in some of the definitions hereafter.

- We introduce the model parameters $\alpha_{ij} = n_{ij}/n_i, i \in \mathcal{N}, j \in \mathcal{N}_i$. These parameters can be estimated in real-time from measurements (e.g. using Kalman filter or maximum likelihood approximation) and the goal is to develop a simple model that can predict their future dynamics without affecting the convexity of the formulations; this can be also done with machine learning techniques and historical data. Note that the parameters can be time varying but they need to be exogenous signals for the optimization.
- New "dummy" control variables $u_{ii}(k_c)$ are introduced $\forall k_c = 0, 1, \ldots, K_c - 1$, that restrict the trip completion rates at every region i. Although these variables are not reasonable from a physical point of view, they are required in order for the problem to be convex. A conjecture is that the solution of MPC will always result in $u_{ii}(k_c) = 1, \forall i \in \mathcal{N}, \forall k_c = 0, 1, \ldots, K_c - 1$, but this needs to be validated with the results.
- We approximate the MFDs of the regions with piecewise affine (PWA) functions $G_i^j(n_i)$, $j = 1, 2, \ldots, N_i$, that form a convex set (see e.g. Figure 3 for a case study with 4 regions). Each MFD can be approximated with $j = 1, 2, \ldots, N_i$ affine functions.
- Most importantly, we introduce new decision variables

$$f_{ii}(k_m) = u_{ii}(k_c)\theta_{ii}(k_m)\left(1 - \sum_{j\in\mathcal{N}_i}\alpha_{ij}(k_m)\right)\frac{P_i(n_i(k_m))}{L_i}, \quad i \in \mathcal{N} \tag{12}$$

$$f_{ij}^h(k_m) = u_{ih}(k_c)\sum_{h\in\mathcal{N}_i}\left(\theta_{ij}^h(k_m)\alpha_{ij}(k_m)\right)\frac{P_i(n_i(k_m))}{L_i}, \quad i, j \in \mathcal{N}, h \in \mathcal{N}_i \tag{13}$$

$$f_{hj}^i(k_m) = u_{hi}(k_c)\sum_{h\in\mathcal{N}_i}\left(\theta_{hj}^i(k_m)\alpha_{hj}(k_m)\right)\frac{P_h(n_h(k_m))}{L_h}, \quad i, j \in \mathcal{N}, h \in \mathcal{N}_i \tag{14}$$

  that help convexify the equations. In equations (12)–(14) the variables $\theta_{ii}(k_m)$, $\theta_{ij}^h(k_m)$, $\alpha_{ij}(k_m)$ are considered time varying exogenous signals and as a result the nonlinearity of the problem comes from the product of the control inputs $u_{ih}(k_c)$ with the MFD functions $P_i(n_i(k_m))/L_i$. The control variables have the property that they are bounded between 0 and 1 and the MFDs that can be approximated by PWA functions. As a result, we are looking for an optimal solution within a convex set, and in this particular case the product can be convexified by introducing the new variables (see Gomes and Horowitz (2006) for some theoretical analysis of a similar convexification in a ramp metering control problem).

  Once the optimal solution is computed then there is a unique transformation of the new variables $f_{ii}(k_m)$, $f_{ij}^h(k_m)$, $f_{hj}^i(k_m)$ to the original control variables $u_{ii}(k_c)$, $u_{ih}(k_c)$, $u_{hi}(k_c)$. This is a modelling trick that allows us to simplify the problem without loosing any accuracy in the dynamics.

2.3 Convex model predictive control (CMPC)

The assumptions outlined above are reasonable approximations/simplifications of the nonlinear model in order to derive a convex formulation that can be used for online MPC. In the sequel we formulate two different versions of the convex model predictive control (CMPC) problem, depending on whether the model keeps track of the origin-destination information of vehicles or not. The models require different online data (as they carry different level of information, i.e. state and demand trajectories ij instead of only i), but under certain assumptions can provide the same optimal solution for the control variables.

2.3.1 CMPC with OD information

The derived convex optimization problem that approximates the original system and can be solved online is as follows

$$\underset{\substack{n_{ii}(k_{\mathrm{m}}),n_{ij}(k_{\mathrm{m}}),\\ f_{ii}(k_{\mathrm{m}}),f_{ij}^{h}(k_{\mathrm{m}}),f_{hj}^{i}(k_{\mathrm{m}})}}{\text{minimize}} \quad \Theta(\mathbf{n}(K_{\mathrm{m}})) + \sum_{k=0}^{K_{\mathrm{m}}-1} \Phi(\mathbf{n}(k)) \tag{15}$$

subject to

$$n_{ii}(k_{\mathrm{m}}+1) = n_{ii}(k_{\mathrm{m}}) + T_{\mathrm{m}}\left(q_{ii}(k_{\mathrm{m}}) - f_{ii}(k_{\mathrm{m}}) - f_{ii}^{h}(k_{\mathrm{m}}) + f_{hi}^{i}(k_{\mathrm{m}})\right), \quad i \in \mathcal{N} \tag{16}$$

$$n_{ij}(k_{\mathrm{m}}+1) = n_{ij}(k_{\mathrm{m}}) + T_{\mathrm{m}}\left(q_{ij}(k_{\mathrm{m}}) - f_{ij}^{h}(k_{\mathrm{m}}) + f_{hj}^{i}(k_{\mathrm{m}})\right), \quad i \in \mathcal{N}, j \in \mathcal{N} \setminus i \tag{17}$$

$$0 \leq f_{ii}(k_{\mathrm{m}}) \leq G_{i}^{l}(k_{\mathrm{m}}), \quad i \in \mathcal{N}, l \in N_i \tag{18}$$

$$0 \leq f_{ij}^{h}(k_{\mathrm{m}}) \leq G_{i}^{l}(k_{\mathrm{m}}), \quad i,j \in \mathcal{N}, h \in \mathcal{N}_i, l \in N_i \tag{19}$$

$$0 \leq f_{hj}^{i}(k_{\mathrm{m}}) \leq G_{i}^{l}(k_{\mathrm{m}}), \quad i,j \in \mathcal{N}, h \in \mathcal{N}_i, l \in N_i \tag{20}$$

$$n_i(k_\mathrm{m}) = \sum_{j\in\mathcal{N}} n_{ij}(k_\mathrm{m}), \quad i \in \mathcal{N} \tag{21}$$

$$0 \le n_i(k_\mathrm{c}) \le n_{i,\max}, \quad i \in \mathcal{N} \tag{22}$$

$$k_\mathrm{m} = 0, 1, \dots, K_\mathrm{m} - 1 \tag{23}$$

2.3.2 CMPC without OD information

Moving one step forward with our approximation, the new model does not need to keep track of the OD information (aggregated information about each region may be sufficient for control purposes). Hence, by adding all the states n_{ij} and n_{ii} for each region i we get a convex model that does not consider OD data but only aggregated demands in the region level. In that case, the derived convex optimization problem that approximates the original system and can be solved online is as follows

$$\underset{\substack{n_i(k_\mathrm{m}), f_{ii}(k_\mathrm{m}),\\ f_{ij}^h(k_\mathrm{m}), f_{hj}^i(k_\mathrm{m})}}{\text{minimize}} \quad \Theta(\mathbf{n}(K_\mathrm{m})) + \sum_{k=0}^{K_\mathrm{m}-1} \Phi(\mathbf{n}(k)) \tag{24}$$

subject to

$$n_i(k_\mathrm{m}+1) = n_i(k_\mathrm{m}) + T_\mathrm{m}\left(q_i(k_\mathrm{m}) - f_{ii}(k_\mathrm{m}) - f_{ii}^h(k_\mathrm{m}) + f_{hi}^i(k_\mathrm{m}) - f_{ij}^h(k_\mathrm{m}) + f_{hj}^i(k_\mathrm{m})\right), \quad i \in \mathcal{N} \tag{25}$$

$$0 \le f_{ii}(k_\mathrm{m}) \le G_i^l(k_\mathrm{m}), \quad i \in \mathcal{N}, l \in N_i \tag{26}$$

$$0 \le f_{ij}^h(k_\mathrm{m}) \le G_i^l(k_\mathrm{m}), \quad i, j \in \mathcal{N}, h \in \mathcal{N}_i, l \in N_i \tag{27}$$

$$0 \le f_{hj}^i(k_\mathrm{m}) \le G_i^l(k_\mathrm{m}), \quad i, j \in \mathcal{N}, h \in \mathcal{N}_i, l \in N_i \tag{28}$$

$$0 \le n_i(k_c) \le n_{i,\max}, \quad i \in \mathcal{N} \tag{29}$$

$$k_m = 0, 1, \ldots, K_m - 1 \tag{30}$$

where $\Theta(\cdot)$, $\Phi(\cdot)$ are any convex functions (e.g. $\Theta = \Phi = \sum_{k=0}^{K_c-1} \sum_{i\in\mathcal{N}} n_i(k)$ represents the total delay, but also quadratic functions can be used, e.g. $\Phi = \sum_{k=0}^{K_c-1} \mathbf{n}^\intercal(k)\mathbf{Q}\mathbf{n}(k)$, where $\mathbf{Q}$ is an appropriate weighting matrix). Note that all the constraints of this problem are linear and as a consequence the computational requirements are quite low, even for a network with many regions and large prediction horizons.

3 Preliminary simulation results

This Section presents the simulation results obtained for the described methodologies. The simulation model (plant) is the nonlinear model presented in Section 2. The test case consists of a network partitioned in 4 regions (Figure 2). It is a replica of the network used in Kouvelas *et al.* (2015, 2016) and corresponds to a part of the CBD of Barcelona in Spain. The partitioning of the network into homogeneous regions has been done by use of the methodology described in Saeedmanesh and Geroliminis (2016). Figure 3 presents the MFDs of the 4 regions from data obtained from a microsimulation model in Kouvelas *et al.* (2015, 2016). The red lines present the PWA approximation of the MFDs with the affine functions employed in the convex

Figure 2: Test case network: CBD of Barcelona partitioned into 4 homogeneous regions.

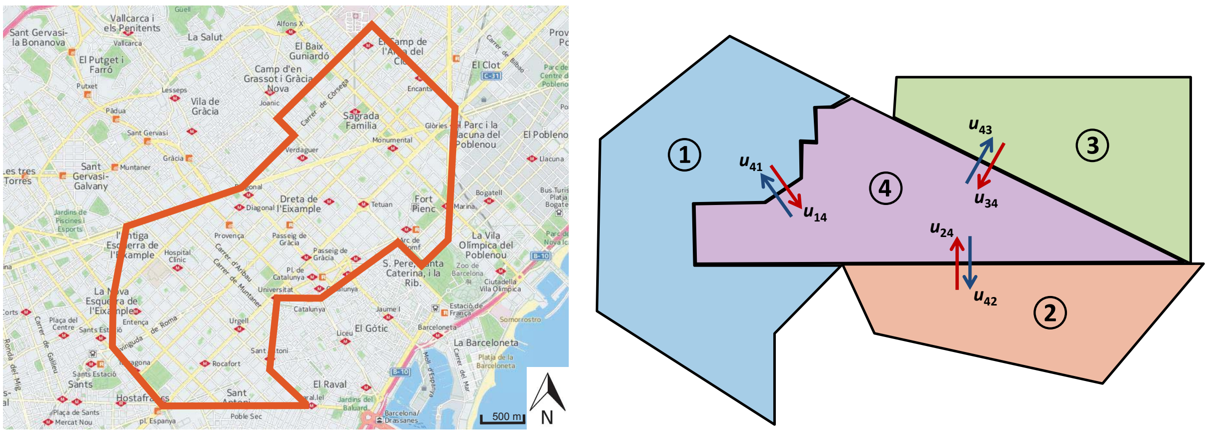
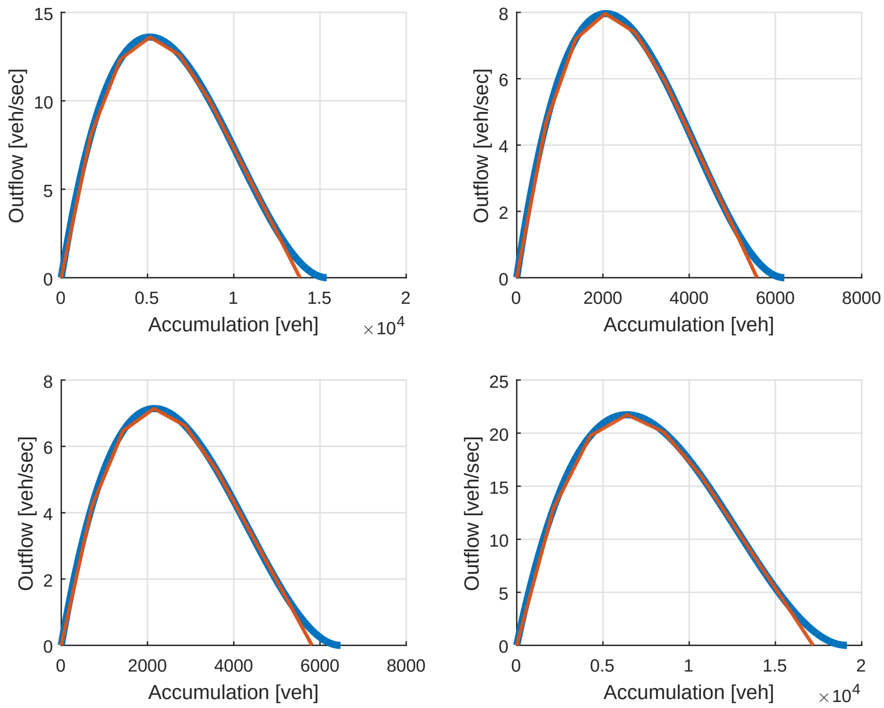


Figure 3: MFDs for the 4 regions of the case study network (blue); piecewise affine approximation of the MFDs to be used in convex MPC (red).



approximation (6 pieces have been used for each MFD here). The approximation is pretty accurate and this relaxation should not cause any problems in the optimization procedure (in that respect the nonlinear model and PWA approximation are almost identical; note also that this approximation can be done with many more lines than presented here without hardening significantly the computations of the convex solver).

First we do a comparison of the plant and the convex models presented in Section 2.3. Figure 4 presents a demand scenario (i.e. generated vehicles per time unit for all the simulation horizon) for the case study with 4 regions (e.g. 4×4 OD matrix). For this demand, Figure 5 displays the evolution of accumulations n_{ij} for the plant and for the model used for CMPC with OD information (Section 2.3.1). Equivalently, Figure 6 displays the evolution of accumulations n_i for the plant and the CMPC model without OD information (Section 2.3.2), which actually corresponds to the estimation of n_i used within the MPC framework. The prediction horizon for both linear models is 12 times higher than the sample time of the plant (e.g. $k_\mathrm{m} = 12k_\mathrm{c}$). The trajectories of the accumulations n_i and n_{ij} demonstrate that both these models can be used to approximate the original nonlinear one (for small prediction horizons). They are both quite accurate representations of the original system, thus appropriate to be used for the convex MPC framework.

Figure 4: Traffic demand for the four regions and all simulation horizon (4×4 OD matrix, i refers to origin and j to destination).

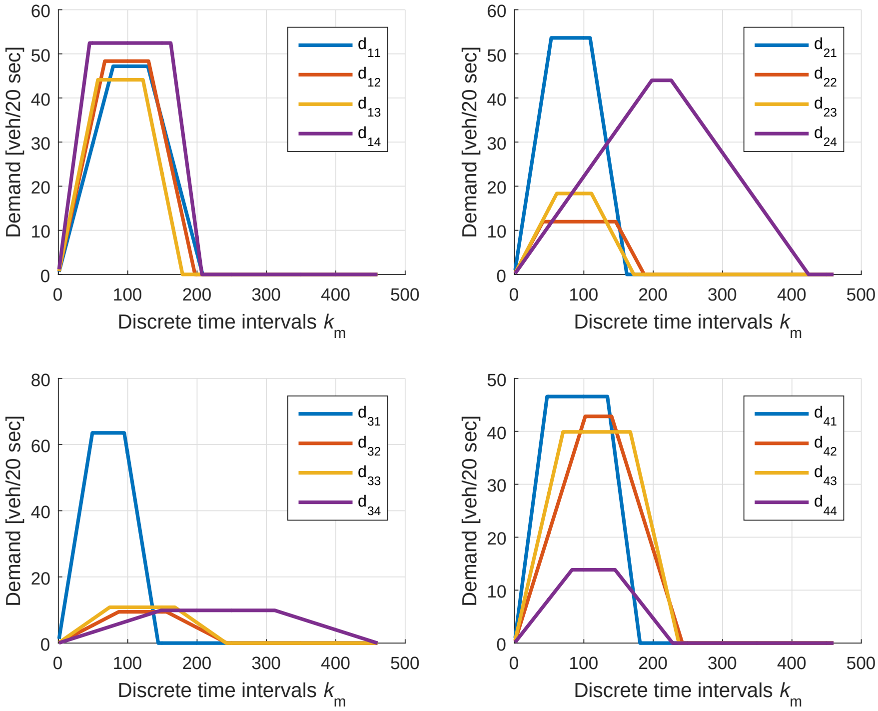


Figure 5: Vehicle accumulations n_{ij} for the plant (solid lines) and the convex model with OD information (dashed lines) when applied for 12 steps of prediction ($k_m = 12k_c$).

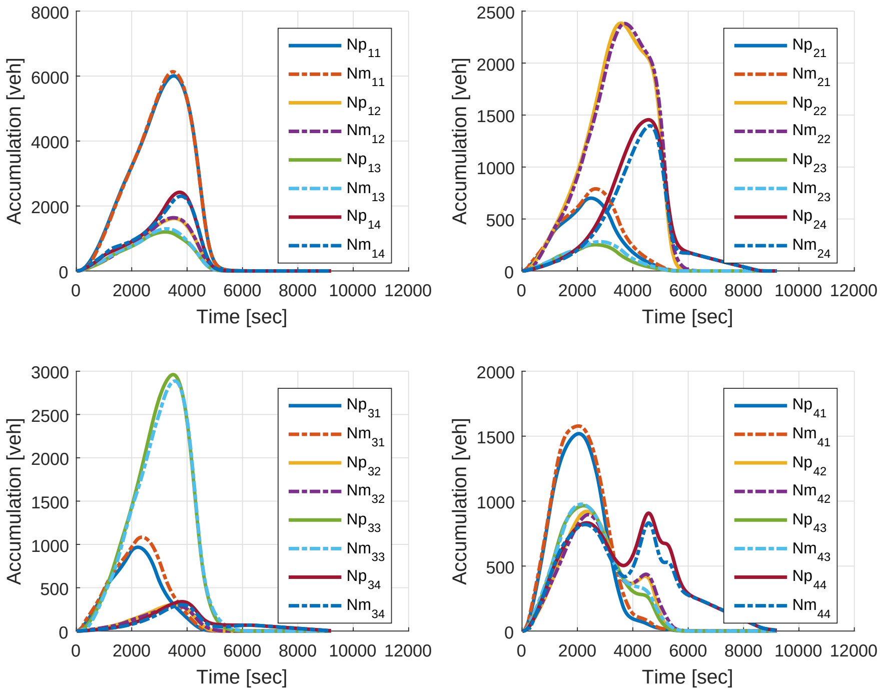
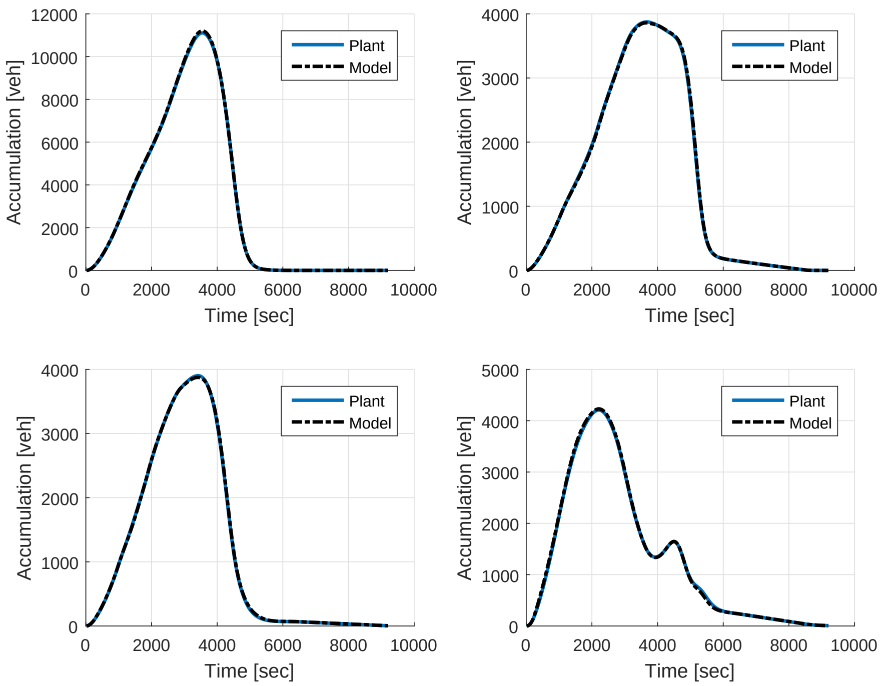


Figure 6: Vehicle accumulations n_i for the plant (blue lines) and the convex model without OD information (dashed black lines) when applied for 12 steps of prediction ($k_\mathrm{m} = 12k_\mathrm{c}$).

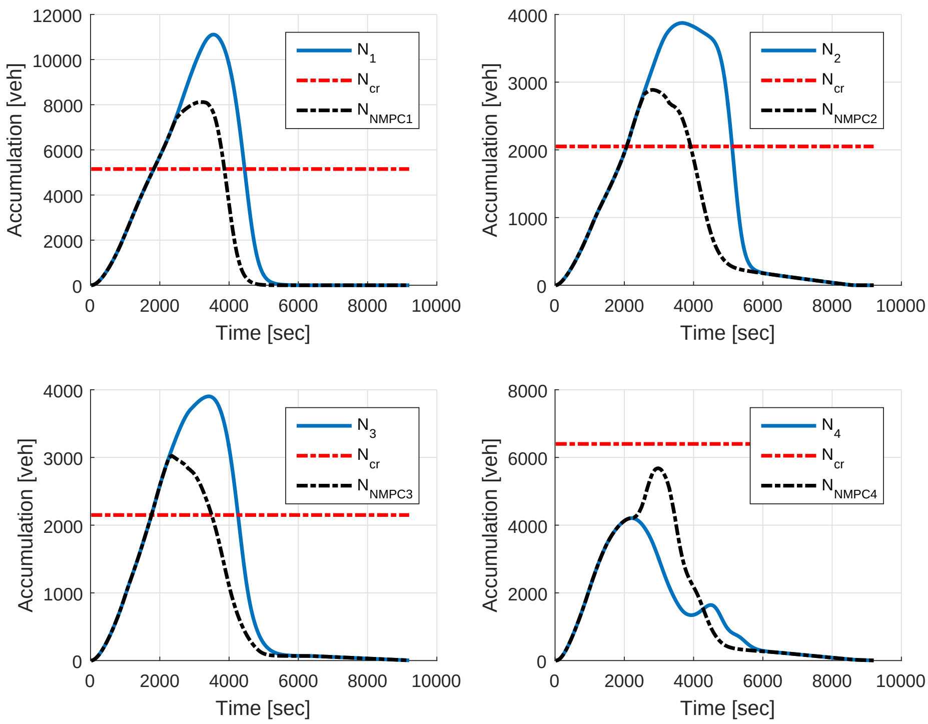


Finally, Figure 7 displays the results for the accumulations when the nonlinear MPC is applied to control the transferring flows between regions. The objective of the controller is to maximize the summation of all trip completions over time in the network (i.e. $\sum_i \sum_{k_\mathrm{m}} M_{ii}(k_\mathrm{m})$). It is clear that the controller improves the traffic states of the system and the area between the blue and the dashed black lines corresponds to the total delay improvement in the system. These results are quite promising and preliminary results for the CMPC (not presented here) show that we can achieve almost the same level of improvement by using the two convex approximations of the model described in Section 2.3.

4 Conclusions

A convex formulation is derived for solving the perimeter control problem in multi-region cities. The originally nonlinear system is relaxed and approximated by a simplified linear model that under certain assumptions can track the behaviour of the multi-region system. The new model requires less information in terms of real-time measurements (e.g. traffic states, OD demands) and as a convex problem it guarantees optimality and fast convergence of the solver. The simulation results need to be further investigated in order to assess how close is the solution

Figure 7: Vehicle accumulations n_i for the 4 regions for no control case (blue lines) and nonlinear MPC (dashed black lines) when applied for 12 steps of prediction ($k_\mathrm{m} = 12k_\mathrm{c}$).



of the convex MPC to the benchmark, which consists of the nonlinear MPC problem with full information about demands and feedback measurements. Note that the benchmark approach in a field implementation would require real-time measurements (or estimates) of n_{ij} and d_{ij}, while the second version of CMPC requires only region-based measurements (i.e. n_i and d_i) and not detailed origin-destination data.

Future work will deal with the development of a more solid methodology for estimating the model parameters α_{ij} (e.g. online Kalman filter), as the values of these parameters are crucial for the optimization horizon. These parameters can be estimated at every control cycle and then considered constant for all the optimization horizon of the MPC but maybe this is not sufficient for the improvement of the system. Simple estimation/prediction techniques can be used to enhance the knowledge for this parameters and help the convex problem to track the nonlinear dynamics in a better way. Investigations about different convex objective functions for the MPC is also another research topic. The proposed methodology needs to be evaluated for different realistic objective functions and demand profiles. Finally, another research direction is to use perimeter control as a first-level controller in cities (as it deals with zone interactions) and develop a second-level of distributed control (e.g. Kouvelas *et al.* (2014)) for optimizing locally. The combination of the two provides a hierarchical control scheme that could potentially be more efficient in alleviating traffic congestion in cities, but this needs to be further investigated.

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