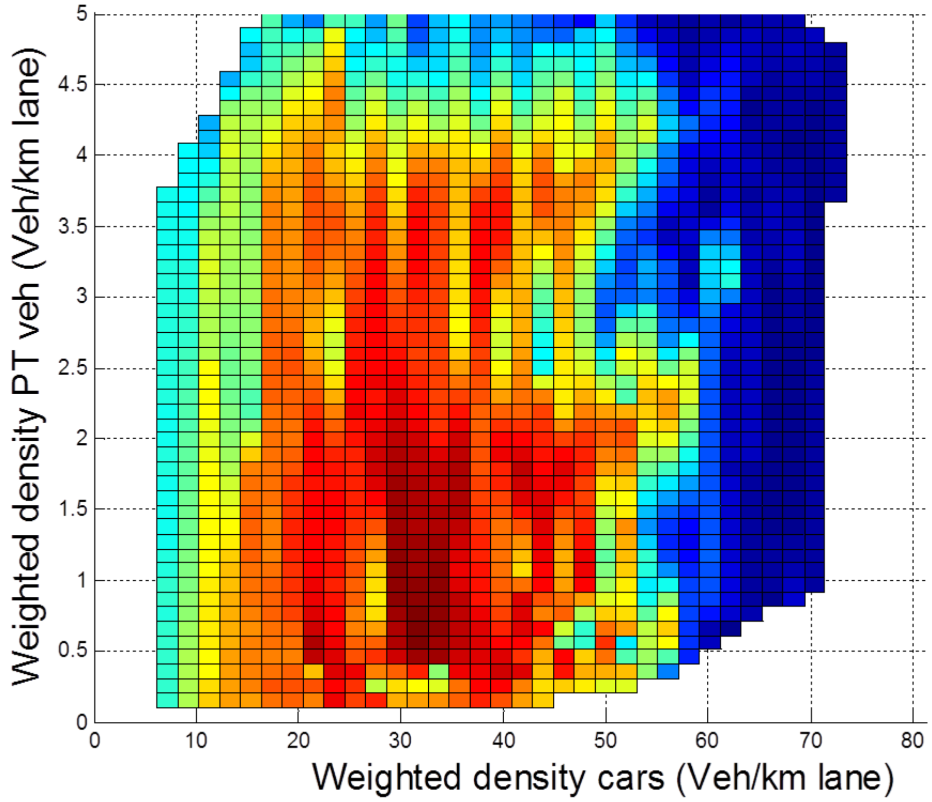

Analysis of space allocation strategies between buses and cars with the multimodal MFD

DRAFT VERSION

Javier Ortigosa, IVT – ETHZ
Nan Zheng, LUTS – EPFL
Monica Menendez, IVT – ETHZ
Nikolas Geroliminis, LUTS – EPFL

Conference paper STRC 2015

STRC

15th Swiss Transport Research Conference
Monte Verità / Ascona, April 15-17,2015

Analysis of space allocation strategies between buses and cars with the multimodal MFD

Javier Ortigosa, IVT – ETHZ
javier.ortigosa@ivt.baug.ethz.ch

Nan Zheng, LUTS – EPFL
nan.zheng@.epfl.ch

Monica Menendez, IVT – ETHZ
monica.menendez@ivt.baug.ethz.ch

Nikolas Geroliminis, LUTS – EPFL
nikolas.geroliminis@epfl.ch

Abstract

In this paper we are studying the trade-off of urban space devoted to cars and buses (or trams) in two cities: Zurich and San Francisco. We employ 3D vehicle Macroscopic Fundamental Diagrams (3D-vMFD) which consist of creating the MFDs for vehicles with several public transport frequencies. We analyze the percentage of dedicated bus lanes in a city. When there are more dedicated bus lanes, the car performance worsens and the transit performance improves. The data is generated with two microsimulation models, VISSIM for Zurich, and Aimsum for San Francisco. The ultimate purpose of this paper is to find the optimal percentage of dedicated bus lanes in a city which optimizes the rate of passengers finishing their trips (either by car or by bus). We present this optimization methodology which can help urban planners to find the most efficient share of space for transportation modes in cities.

Keywords

Traffic operations – MFD – 3D-vMFD – Dedicated lane – Mixed lane – Public Transport.

1. Introduction

In current cities, road space is scarce. On the one side, individual transport does not seem to decrease, and urban population is increasing. On the other side, cities are implementing more and more actions towards recovering road space for more sustainable transport modes, e.g. public transport, cycling, and walking. This competition forces planners to be more and more efficient with the existing road space.

Macroscopic network-wide traffic models were first proposed by Godfrey (1969), Mahmassani et al. (1987), and Daganzo (2007). Later, Geroliminis and Daganzo (2008) and Daganzo and Geroliminis (2008) demonstrated that a relationship between trip-completion rate and the number of vehicles circulating in the network exists, and also an aggregate relationship between average network flow and density on links, known as the Macroscopic Fundamental Diagram or MFD. This relationship exists and it is independent of the OD patterns when flows are distributed homogeneously across the network. When that is not fulfilled, the aggregate flow-density relationships present more scatter and exhibit lower trip completion rate than the MFD, which is the upper bound (Mazoumian et al., 2010; Geroliminis and Sun, 2011; Daganzo et al., 2011; Mahmassani et al., 2013; Knoop et al., 2013). The MFD has been proved to be particularly useful for traffic control scheme strategies (Geroliminis and Boyaci, 2012; Koop et al., 2012; Keyvan-Ekbatani et al., 2013; Haddad and Shraiber, 2014; Haddad and Geroliminis, 2012; Geroliminis et al., 2013; Yildirimoglu and Geroliminis, 2014; Ramenazi et al., 2014). Many of these research works divide the whole urban area in smaller regions with homogeneous traffic and a MFD associated to them, this way, the traffic control problem becomes more complex as every region receives and sends trips to the others.

Building on the knowledge of the single-mode MFD, developing and understanding the dynamics of multimodal networks is starting to be more relevant in research. On these macroscopic models, there has been several research works that aim at analyzing the trade-off between transportation modes, e.g. Gonzales et al. (2011). Zheng et al. (2013) and Zheng and Geroliminis (2013) studied the mode choice at a macroscopic level with a direct feedback on the traffic performance and they propose an optimization methodology based on the distribution of urban space. Ultimately, Geroliminis et al. (2014) proposed and investigated the existence and the properties of a three-dimensional vehicular MFD (3D-vMFD) relating the accumulation of cars and buses, and the total circulating vehicle flow in the network based on simulated data. In Geroliminis et al. (2014) the network offers the same priorities and road space availability to both modes. However, this in reality would make public transport systems very unreliable as their performance would depend on the traffic situation. City and traffic planners have been giving a higher priority of public transport systems to keep a steady performance, to discourage individual road transport, and to gain more users. Many cities provide dedicated bus lanes to ensure that buses do not compete for road space with cars. In other cases, or in addition to them, cities, as it is the case of the City of Zurich, also give signalization priority to public transport convoys when they approach an intersection.

In this paper, we are analyzing specifically how the presence of dedicated bus lanes and signal priority systems affects the 3D-vMFD as we want to further understand the macroscopic performance of a city with these existing conditions. In this paper we compare with simulation methods the 3D-vMFDs with and without dedicated bus lanes for the City of Zurich and San Francisco. Moreover, we present an optimization method based on Zheng and Geroliminis (2013) and Zheng (2014) to provide us with the optimal number of dedicated bus lanes that a city should have to optimize passenger travel times.

2. The Zurich case

In this first part of the paper, we analyze the space trade-off in the inner city of Zurich. We employ a microsimulation model of 2.6 km^2 area. The demand corresponds to the 5–6 pm afternoon peak on a working day. To obtain the whole MFD we scale up the demand with factors ranging from 0.4 to 2.0. This model has been calibrated using flows from vehicle counts in 2010 (Heimgartner 2012; Menendez and Ge 2012). The unimodal MFD of Zurich was obtained with this procedure by Ortigosa et al. (2014).

In the inner city of Zurich there is a high presence of public transport (PT), most of it, trams. 95 % of the lanes in the inner city are dedicated lanes, either for cars (75% of the total) or for trams and buses (18% of the total), only a 5% is actually mixed. In our simulation model, we consider 19 public transport lines. We consider several service-headway scenarios to find different accumulation values of PT vehicles, particularly 2 min, 4 min, 8 min, and 16 min. The microsimulation model emulates also the priority system for public transport that Zurich signals have. Unfortunately, when the public transport headways are very short, the system cannot cope with such amount of vehicles and some intersections are blocked.

We collect the average circulating traffic flow (veh/h/lane) and the average vehicular densities (veh/km/lane) from all network links, every 5min. The 3D-vMFD of network with the dedicated lanes is presented in Figures 1a-1c. In Figures 1d-1f we have repeated the same simulation but allowing both cars and public transport to circulate in all lanes. Note that although the routing is still maintained, i.e. cars will not take new routes even if now they could; we see how in the streets where there were both cars and buses/trams in separated lanes, now they mix. We also consider trams to have the same properties as buses. As a general remark, our results confirm that 3D-vMFD exists in multimodal networks with shared and dedicated lanes, which are consistent with what have been reported in literature.

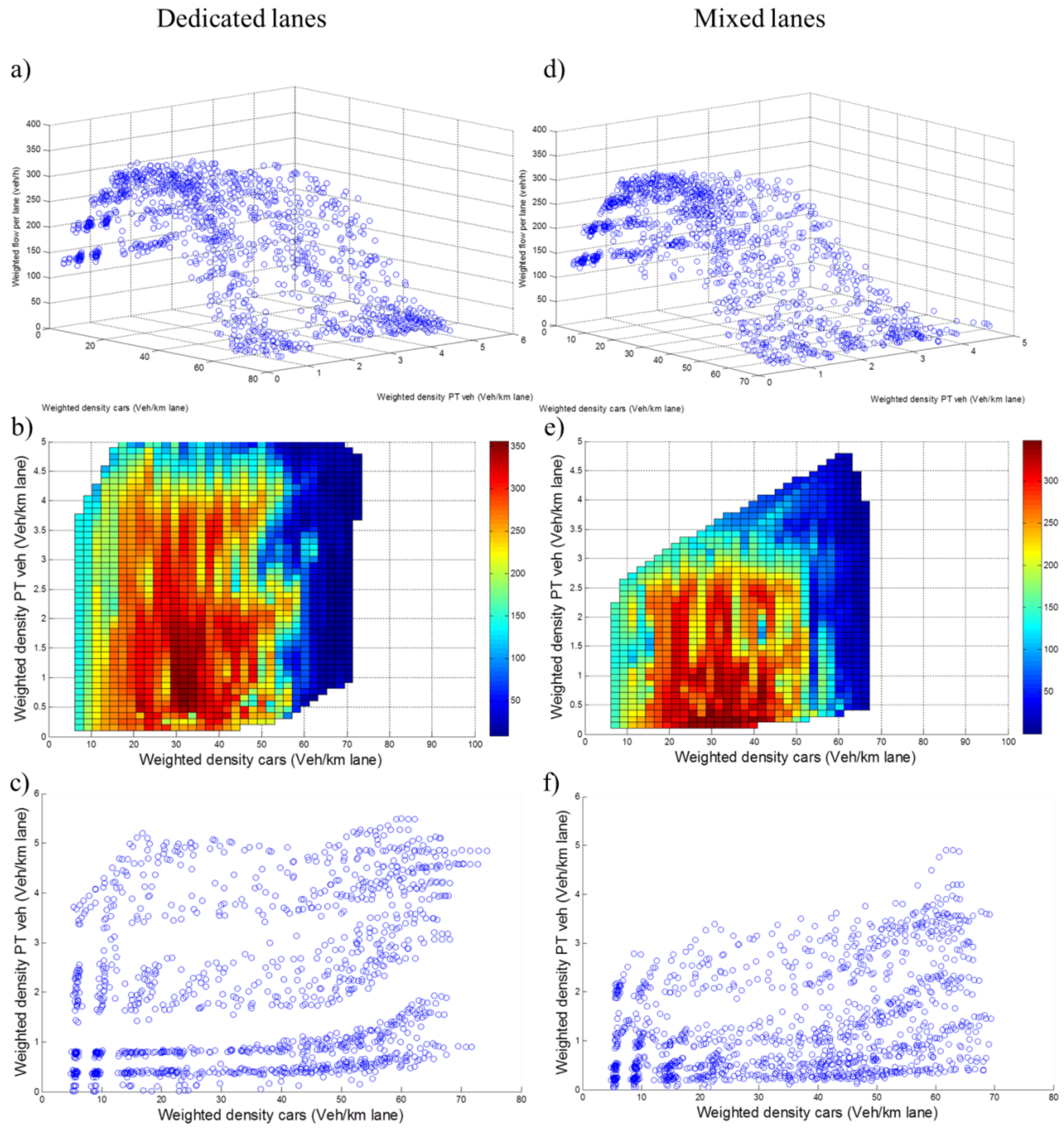

Figure 1. 3D-vMFD representations for the City of Zurich: a-c) dedicated lanes, d-f) mixed lanes.

Let us have a closer look on the results. Figures 1a-1c show the case of dedicated lanes. The total circulating capacity of vehicles is approximately 340-350 veh/h/lane. This flow capacity reaches its maximum when the service frequency of the PT modes is the lowest: for car densities ranging 25-40 veh/km/lane, and for PT densities between 0.3 and 0.8 veh/km/lane. We observe that, as the PT density increases, the maximum flow decreases, for example, the maximum flow of vehicles ranges between 290 and 310 veh/h/lane when the PT density is around 3.7 veh/km/lane. Figures 1d-f depict the 3D-vMFD when all the lanes are mixed. The maximum flow values are slightly higher than the previous case, ranging between 345 and 355 veh/h/lane. These values can be found at slightly higher car densities, and lower PT densities between 0.2 and 0.5 veh/km/lane. When the public transport density becomes higher, the same pattern is repeated. The maximum flow is higher than the previous cases (ranging between 300 and 320 veh/h), reached at higher car densities and considerably lower public transport densities (approximately 2 veh/km).

What we find interesting by comparing the two cases is that the car performance is somewhat better when all lanes are mixed than when they are not. Instead, the PT performance seems to lower and presents lower PT densities (see Figure 1c and 1f for a comparison). Note that these results might be limited with the priority system, as in reality they might not function so ideally that public transport frequencies are maintained so high.

Let us now consider how the 3D-vMFD looks when the priority signal system is not activated. We simulate again the scenario with dedicated bus and tram lanes but employing a fixed signaling system.

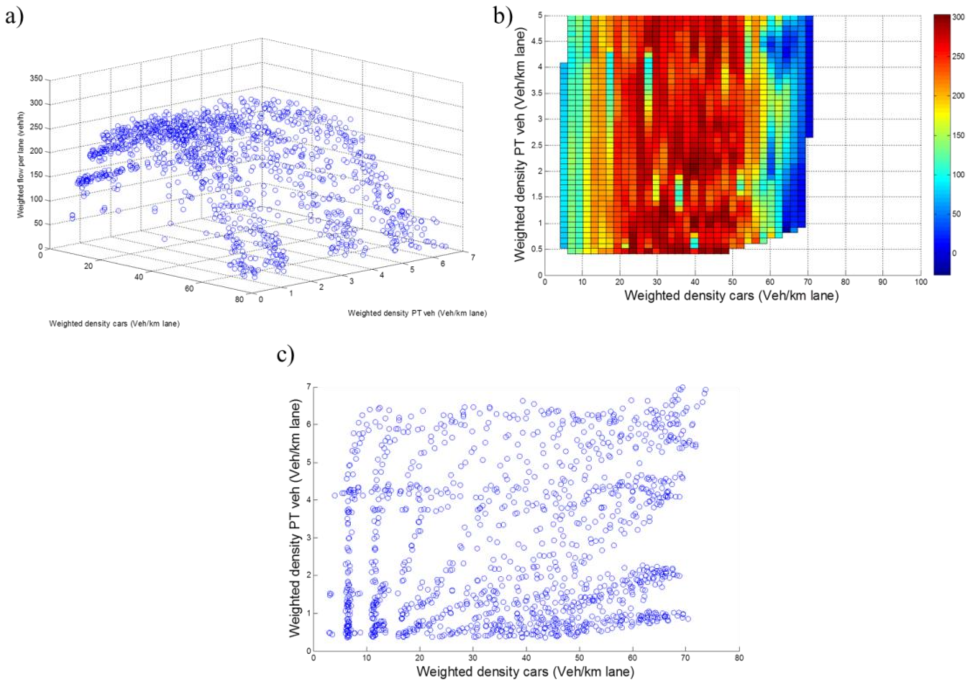

Figure 2. 3D-vMFD for the City of Zurich with dedicated lanes and without the priority signal system.

Figure 2 illustrates that the maximum flow capacity of vehicles does not change with the increase of public transport frequency. That makes sense because most of the lanes are dedicated and the same capacity (via green-signal time setting) is given at intersections regardless of the amount and type of vehicles. Looking at the actual values, we see how the maximum capacity is in this case lower than for the cases considered in Figure 1. The values range between 300 and 310 veh/h/lane. One explanation is that a system without priority is inefficient and less adaptable to traffic flows. Another possible cause is the fact that we employ an older version of the network where there are some main intersections that do not have signal control (e.g. the intersection of Central and Bellevue). Normally such non-signalized intersections accommodate less flow capacity. Furthermore, by comparing Figure 2c and Figure 1c, we see how the range of PT accumulations change in the no-priority case which is slightly wider. Moreover, Figure 1c shows that a network with dedicated lanes keeps much more consistency in PT operation, as the density values of PT remain nearly constant. One can perfectly identify the classical 2D-MFDs at different PT accumulation values, reflecting the different headway scenarios considered. On the other hand, in Figure 2c, the densities of both modes correlate and change over traffic states, showing high variability.

3. The San Francisco case

To verify the observations from the 3D-vMFDs of the Zurich case study, we investigate in this section the traffic performance in another well developed and calibrated multimodal urban network: the downtown San Francisco network. We have created the 3D-vMFDs of the network employing the simulation data from the microsimulator Aimsum. This part of the discussion will be based on the work of Geroliminis et al. (2014) who presented the existence and property of a 3D-vMFD for mixed–lane case. Here we extend the study to dedicated-lane case, and present and compare results on both the 3D-vMFD with mixed lanes and the one with dedicated lanes.

The test site is a 3 km^2 area of Downtown San Francisco, where cars and buses share the network, including about 100 intersections and 430 links of total 100 lane-km. The number of lanes varies from 2 to 5 lanes and the free flow speed is around 45 km/h. There is no any preferential treatment of buses, e.g. transit signal priority (TSP). Bus stops are mostly in-lane of traffic and create queues of cars during service stopping. The traffic flow in the network comprises two vehicle classes: passenger cars and buses. Bus routes and frequencies for lines in the studied network have been obtained from the San Francisco Municipal Transportation Agency (SFMTA). We simulate a 4-hour morning peak traffic. The total demand has a typical trapezoid-shape morning peak profile. While for buses, they are defined by service frequency which is the same as in the Zurich case study. The headway outside peak hour (about 2h) ranges between 4min-20min, while during peak hour (about 3h) between 2min-10min. The average dwell time of buses is defined as well, ranging between 25s to 60s, and a normal distribution.

In Figure 3a-3c, we display the resultant scatter plot and the contour plot of the 3D-vMFD for an extended version of the San Francisco network, where dedicated bus lanes are created for the center region on the busies roads. We reproduce the same plots for the mixed-lane case which was reported in Geroliminis et al. (2014) for comparison. As a first remark, similar patterns can be observed between the dedicated-lane case and the mixed-lane case as in the Zurich case: (i) the dedicated-lane case maintains higher flow when the density of buses is higher, and the difference in maximum flow is not significant as the accumulation of bus increases; (ii) fixing a certain bus density value, an MFD – shape relation can be observed; and (iii) the optimal network operation is achieved at very low bus accumulation. For (iii), as Geroliminis et al. (2014) shows, it is not valid anymore as long as passenger flow is instead considered.

What is interesting is that, in the San Francisco network, (i) vehicle flow drops much faster as the density of buses increases (from 250 veh/h/lane to 100 veh/h/lane when PT density increases from 0.5 veh/km/lane to 4.5 veh/km/lane; see Figure 3e), indicating that buses have more sensitive interactive influence on the overall performance, and (ii) the variation in mode composition (e.g. the ratio between car and bus densities) is much smaller than in the Zurich case, indicating a more reliable operation in this network. The reasons can be attributed to the difference in signal settings, topology and driving behaviors. We will investigate more and report this part as future work.

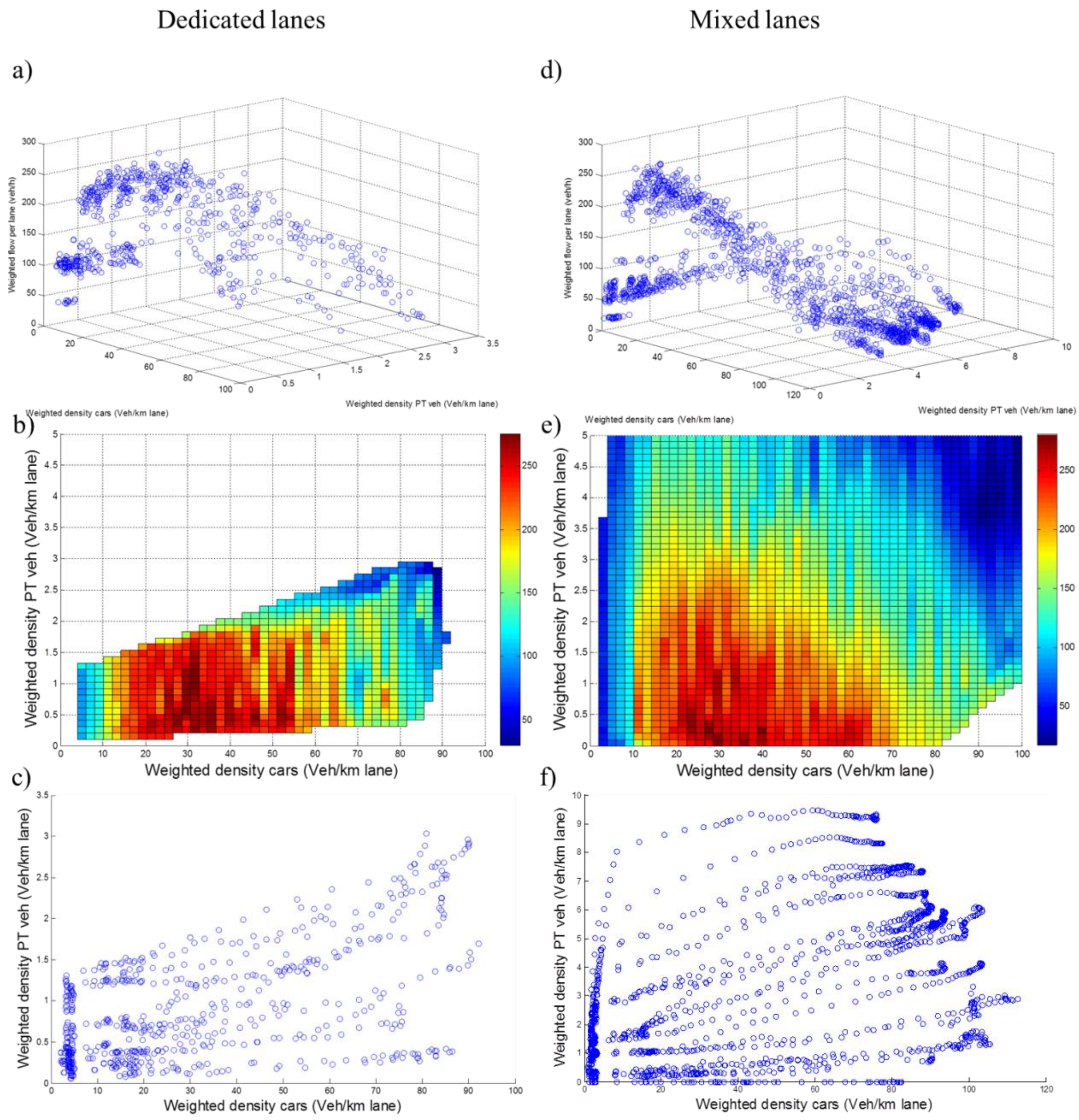

Figure 3. 3D-vMFD representations for San Francisco City: a-c) dedicated lanes, d-f) mixed lanes (Geroliminis et al., 2014).

Note that there are empty spaces in the plots for the dedicated-lane case, as we do not yet obtain enough data to complete all the PT scenarios. One physical explanation is that San Francisco has more lanes in each street in comparison to the inner city of Zurich where most of the streets have only one lane for car traffic. Furthermore, buses stop in a car traffic lane in San Francisco network, blocking cars to go through, whereas for the case of Zurich, that does not always happen. Nevertheless, the current 3D-vMFD enables us the comparison between the two lane-allocation cases, and the two different networks, and these scenarios where buses have extremely high presence are not realistic in field operation.

4. The optimization process

Motivated by the findings in Section 3, we are interested in developing a tool to decide what percentage of the lanes in a network should be given to dedicated bus lanes, such that the total system performance, e.g. total vehicle flow or passenger flow, is maximized. In this section, we propose and detail the methodology that we will follow to find the optimal percentage of bus lanes that a city should have based on the MFDs of that urban area. This methodology is an extended version of the particular case (2 regions) presented by Zheng (2014). Results on space allocation optimization will be presented in a later version of the paper.

While for the generalized formulae a reader should refer to Zheng (2014), let us here consider a city divided into 2 regions: 1) center and 2) periphery; with two transport modes: cars (c), and buses (b). Our study focus will be given on region 1 (city center) where congestion is sever and space is limited, e.g. it is impossible to allocate every link a bus lane. Denote π the fraction of road space in this region devoted to dedicated bus lanes, while the (1-π) percentage is uniquely utilized by cars. Thus we have two reservoirs of dedicated lanes. In each of these reservoirs and the periphery region 2, we assume congestion is distributed homogeneously and we can estimate the MFDs. Note that, this MFD can be a 2D-MFD or 3D-MFD. Given the total demand, with the MFD well defined for each mode and region, a system model will reproduce the change of accumulation of vehicles, $n(t)$, and the outflow or trip completion rate, $O(t)$, over time and estimate the resultant traffic performance under a certain allocation plan (π, 1-π). The problem to solve is then a road space allocation optimization to minimize the total passenger hours travelled (PHT) over time for all travelers. Mathematically, the optimization problem is formulated as follows:

$$\begin{aligned}\min_{\pi} Z &= \min_{\pi}\left[\sum_t \mathrm{PHT}_{t,c}\,(\pi) + \sum_t \mathrm{PHT}_{t,b}\,(\pi)\right] \\ &= \min_{\pi}\left[\sum_t \left(n_1^c(t)\cdot ob_1^c + OB_1^b(t)\right)\cdot T\right]\end{aligned} \tag{1}$$

where $n_1^c(t)$ is the car accumulation in region 1 for time step *t*, ob_1^c is the occupancy of cars which is a constant (say 1-1.3 passenger per car), $OB_1^b(t)$ is the number of bus passengers in region 1 for time step *t*, and *T* is the duration of the interval. The car accumulation in region 1 for interval *t*+1 will be:

$$n_1^c(t+1) \;= n_1^c(t) + \frac{Q_1^c(t+1)}{ob_1^c} - O_1^c(t) + \; O_{2\to 1}^c(t) \tag{2}$$

where, $Q_1^c(t+1)$ is the car demand generated in region 1 (the total demand $Q_1(t)$ is known and how it is distributed to the two modes will be described shortly), $\frac{Q_1^c(t+1)}{ob_1^c}$ then is the number of generated car trips; $O_1^c(t)$ is the outflow in region 1 (either because they are moving to region 2 or because the trip ends in region 1); and $O_{2\to 1}^c(t)$ is the inflow in region 1 (trips coming from the periphery region 2). The OD demand is also assumed given and we know at the regional level $Q_1(t) = Q_{1\to 2}(t) + Q_{1\to 1}(t)$ (the same applies to region 2).

For the cases of buses, the movement dynamics are expressed in the following:

$$n_1^b(t+1) = n_1^b(t) - O_{1\to2}^b(t) + O_{2\to1}^b(t) \tag{3}$$

We consider there is no generation of buses within zones, as buses serve circular routes in the city, and it is only the passenger demand that changes. To estimate and evaluate the system performance under different allocation schemes, it is indispensable to look at the passenger flow dynamics. After all, smart space allocation strategies should aim at maximizing the total passenger utility, as in our case minimizing the total travel time. Equation 4 shows a simple on-board passenger mass conservation equation that calculates the number of passengers using buses, $OB_1^b(t)$, change over time:

$$OB_1^b(t+1) = OB_1^b(t) + Q_1^b(t+1) - O_{1\to2}^b(t) \cdot ob_1^b(t)$$
$$+O_{2\to1}^b(t) \cdot ob_2^b(t) - OB_1^b(t) \cdot (1-(1-\theta_1)^z) \tag{4}$$

where

$$ob_i^b(t) = \frac{OB_i^b(t)}{n_i^b(t)}, \tag{5}$$

The last summand on the RHS of Equation 4 approximates the passenger trip endings in region 1 as the mean of a Bernoulli trial which is repeated z times for each passenger with probability of success θ_1 -see for more details (Zheng, 2014).

The average travel time of cars, $TT_1^c(t)$, can be estimated as $\bar{L}_{1c}/V_1^c(t)$. $\bar{L}_{1c}$ is the average trip length and $V_1^c(t)$ is the average speed. For buses, the travel time has two compounds: the total time spent at bus stops (total dwell time) $TT_d^b(t)$, and the average travel time of buses excluding TT_d^b, which is given by $\bar{L}_{1b}/V_1^b(t)$. See Zheng (2014) to see how average trip lengths and speeds are calculated. Knowing travel times, we can calculate the utility functions for choosing each mode at the beginning of a trip. We add a term of extra costs $C_1^c(t)$ for cars vs. buses (e.g. parking, tolls, fare), and an on-board spatial discomfort $D_1^b(t)$ for buses, e.g. crowdedness; see Zheng (2014) for further details.

$$U_1^c(t) = -(TT_1^c(t) + C_1^c(t))$$
$$U_1^b(t) = -(TT_1^b(t) + D_1^b(t)) \tag{6}$$

The optimization of Equation 1 is subject to the system dynamics as described above. The optimization problem becomes much more complex when π depends on spatial and temporal patterns, e.g. if there are bus lanes that can be activated or deactivated depending if it is a peak hour. We solve this problem by a non-linear programming method, the sequential quadratic programming (SQP). For details of the SQP method, please refer to Nocedal and Wright (2006). We apply this algorithm for multiple initial values (around 1000) to avoid convergence to local minima, which might be the case for a non-smooth objective function.

The proposed system model reproduces the aggregated traffic dynamics and performance of urban networks. The numerical simulation is able to provide a qualitative evaluation of different space allocation schemes in detailed multimodal environments, such as Zurich and SF networks utilized in the previous two sections. The periphery region 2 can serve as the demand generated at the boundary of the networks, whereas the center region 1 will be the case study networks. The MFDs, network topology, demand profile, mode split, will be the inputs to the system model which can be obtained from the simulations. Executing the optimization procedure, we obtain the optimal π. On-going work calibrates the system model qualitatively so that it can be applicable for the generalized version of the two case study networks.

5. Conclusions

In this paper, we study the space trade-off in cities between cars and public transport vehicles. Both modes compete for the space and their performance is related to it. Our analysis is based on the macroscopic perspective. This work builds on Zheng and Geroliminis (2013), Zheng (2014), and Geroliminis et al. (2014) that analyze the space trade-off between cars and buses employing the Macroscopic Fundamental Diagram (MFD). Particularly, Geroliminis et al. (2014) present the 3D-vMFD which considers also the dimension of the accumulation of public transport vehicles.

The first part of this paper presented the 3D-vMFD of the City of Zurich obtained with the VISSIM microsimulator. The City of Zurich, not only has a high percentage of dedicated bus and tram lanes, but also a priority signal system for public transport vehicles. These two features provide more capacity to public transport in comparison to other cities. We analyzed the current case, but also we generated 3D-vMFDs for the cases of having shared or mixed lanes, and for a fixed signal system. What we see (Figures 1 and 2) is that in the cases of mixed lanes and not having a signal priority, although car performance might not be affected (or even benefited), public transport performance worsens as the accumulations of buses or trams are lower and more scattered.

The second part of this paper compared the 3D-vMFD of the City of San Francisco when lanes are mixed (Geroliminis et al., 2014) and when lanes are dedicated. Despite the topological and behavioral differences between the two cities, we can see how the dedicated-lane case maintains higher flow when the density of buses is higher, and the difference in maximum flow is not significant as the accumulation of bus increases. When we fix a certain bus density value, an MFD –shape relation can be observed. Moreover, the optimal network operation is achieved at very low bus accumulation.

Finally, the third part of this paper proposed an optimization methodology to find the percentage of dedicated bus/tram lanes that provide the minimum passenger travel time. This methodology is presented in Zheng and Geroliminis (2013) and Zheng (2014). In this paper we have adapted it to the case of a city with only two regions, central and periphery, and two transport modes, car and bus.

This is a work in progress. The next steps on this research will be analyzing in more detail the different 3D-vMFD shapes, and applying the optimization methodology.

6. References

Daganzo, C. F. Urban gridlock: Macroscopic modeling and mitigation approaches. *Transportation Research Part B*, Vol. 41, No. 1, 2007, pp. 49-62.

Daganzo, C. F., and N. Geroliminis. An analytical approximation for the macroscopic fundamental diagram of urban traffic. *Transportation Research Part B*, Vol. 42, No. 9, 2008, pp. 771-781.

Daganzo, C. F., V. V. Gayah, and E. J. Gonzales. Macroscopic relations of urban traffic variables: bifurcations, multivaluedness and instability. *Transportation Research Part B*, Vol. 45, No. 1, 2011, pp. 278–288.

Ge, Q., M. Menendez. Final Report of Calibration Study for VISSIM (CSV). *Project report for the Division of Transport*, City of Zurich, 2012.

Geroliminis, N., and B. Boyaci. The effect of variability of urban systems characteristics in the network capacity. *Transportation Research Part B*, Vol. 46, No. 10, 2012, pp. 1607-1623.

Geroliminis, N., and C. F. Daganzo. Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B*, Vol. 42, No. 9, 2008, pp. 759-770.

Geroliminis, N., J. Haddad, and M. Ramezani. Optimal perimeter control for two urban regions with macroscopic fundamental diagrams: A model predictive approach. *Intelligent Transportation Systems, IEEE Transactions, Vol.* 14, No. 1, 2013, pp. 348-359.

Geroliminis, N., J. Sun. Properties of a well-defined macroscopic fundamental diagram for urban traffic. *Transportation Research Part B*, Vol. 45, No. 3, 2011, pp. 605–617.

Geroliminis, N., N. Zheng, and K. Ampountolas. A three-dimensional macroscopic fundamental diagram for mixed bi-modal urban networks. *Transportation Research Part C: Emerging Technologies,* Vol. 42, 2014, pp. 168-181.

Godfrey, J. The mechanism of a road network. *Traffic Eng. Control*, Vol. 11, No. 7, 1969, pp. 323–327.

Gonzales, E., C. Chavis, Y. Li, and C. Daganzo. Multimodal transport in Nairobi, Kenya: insights and recommendations with a macroscopic evidence-based model. In: *The 90th Annual Meeting of the Transportation Research Board*, Washington D.C., 2011.

Haddad, J., and A. Shraiber. Robust perimeter control design for an urban region. *Transportation Research Part B*, Vol. 68, 2014, pp. 315-332.

Haddad, J., and N. Geroliminis. On the stability of traffic perimeter control in two-region urban cities. *Transportation Research Part B*, Vol 46, No. 9, 2012, pp. 1159-1176.

Heimgartner, C. Virtual ITS evaluation establishment of micro-simulation for the city center of zurich. In: *19th ITS World Congress*, Vienna, 2012.

Keyvan-Ekbatani, M., M. Papageorgiou, and I. Papamichail. Urban congestion gating control based on reduced operational network fundamental diagrams. *Transportation Research Part C*, Vol. 33, 2013, pp. 74–87.

Knoop, V. L., S. P. Hoogendoorn, and J. W. Van Lint. Routing strategies based on macroscopic fundamental diagram. In *Transportation Research Record: Journal of the Transportation Research Board, No. 2315*, Transportation Research Board of the National Academies, Washington, D.C., 2012, pp. 1-10.

Knoop, V. L., S. P. Hoogendoorn, and J. W. Van Lint. The impact of traffic dynamics on macroscopic fundamental diagram. *92nd Annual Meeting Transportation Research Board*, Washington, USA, 2013.

Little, J.DC. A proof for the queuing formula: L= λ W. *Operations research*, Vol. 9, No .3, 1961, pp. 383-387.

Mahmassani, H., J. Williams, and R. Herman. Performance of urban traffic networks. In: *Proceedings of the 10th International Symposium on Transportation and Traffic Theory*, Massachusetts, USA, 1987, pp. 1–20.

Mahmassani, H., M. Saberi, and K. Ali Zockaie. Urban network gridlock: Theory, characteristics, and dynamics. *Procedia-Social and Behavioral Sciences*, Vol. 80, 2013, pp. 79–98.

Mazloumian, A., N. Geroliminis, and D. Helbing. The spatial variability of vehicle densities as determinant of urban network capacity. *Philosophical Transactions A*, Vol. 368, 2010, pp. 4627-4647.

Nocedal, J., and S. J. Wright. *Least-Squares Problems*. Springer New York, 2006.

Ortigosa, J., M. Menendez, and H. Tapia. Study on the number and location of measurement points for an MFD perimeter control scheme: a case study of Zurich. *EURO Journal on Transportation and Logistics*, Vol. 3, No. 3-4, 2014, pp. 245-266.

Ramezani, M., J. Haddad, and N. Geroliminis. Dynamics of heterogeneity in urban networks: aggregated traffic modeling and hierarchical control. *Transportation Research Part B*, Vol. 74, 2015, pp. 1-19.

Yildirimoglu, M., and N. Geroliminis. Approximating dynamic equilibrium conditions with macroscopic fundamental diagrams. *Transportation Research Part B*, Vol. 70, 2014, pp. 186-200.

Zheng, N. A Dynamic Network Approach for Multimodal Urban Mobility: Modeling, Pricing and Control. *Doctoral Thesis*. Ecole Polytechnique Federale de Lausanne, 2014.

Zheng, N., and N. Geroliminis. On the distribution of urban road space for multimodal congested networks. *Transportation Research Part B: Methodological*, Vol. 57, 2013, pp. 326-341.

Zheng, N., K. Aboudolas, and N. Geroliminis. Investigation of the existence of city-scale three-dimensional macroscopic fundamental diagrams for bi-modal traffic. *Proceeding of the 16th IEEE Conference on Intelligent Transportation System*, The Hague, The Netherland, 2013, pp. 1029–1034.

Zheng, N., R. Waraich, K. Axhausen and N. Geroliminis, 2012. A dynamic cordon pricing scheme combining the Macroscopic Fundamental Diagram and an agent-based traffic mode. Transportation Research Part A: Policy and Practice, Vol. 46, No. 8, pp. 1291–1303.