# Bid rent model for simultaneous determination of location and rent in land use microsimulations

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## Abstract

A method for (micro)simulation of location choice of households is proposed. The method is based in the bid-action approach for land use modeling, which assumes that rents can be estimated as the expected maximum bid in an auction. The method allows for period-wise simulation of location choice where rents are adjusted depending on the households' perceptions of the market conditions. The location can be modeled both as an auction or as a direct choice, depending on the market conditions. This allows for the simulations of both demand and supply surplus scenarios in a consistent way.

## Keywords

location choice, hedonic rents, equilibrium, auction market, real estate

# 1 Introduction

Residential location choice and real estate prices have been traditionally modeled under two major paradigms: the choice approach and the bid-auction approach. Under the choice paradigm, households select the location that maximizes their utility, with prices being determined exogenously through a hedonic model. The bid-auction approach assumes that real estate good are traded in an auction market, where the best bid for a particular location determines both the located household and the price of the dwelling.

Both bid-auction and choice approaches work under the assumption that prices will be properly estimated only under equilibrium conditions. In the choice case the hedonic approach for modeling prices implicitly assumes that the (equilibrium) market values of the attributes of a location are represented in the parameters of a regression. The bid-auction approach can only determine prices when all households have interacted in all the possible auctions, achieving a state where no household can improve his situation by changing its location.

The underlying equilibrium assumption makes hard to implement either approach directly in a microsimulation context, where equilibrium is never solved but, instead, a dynamic process approximates the equilibrium conditions by simulating all the individual interactions in the market. For operational reasons, microsimulation models usually favor a choice approach, estimating the hedonic price model for a base modeling period and ignoring the equilibrium assumption. This means that prices are insensitive to changes in the market conditions (e.g. income distribution across the population, supply or demand surplus), making the market values of each of the attributes of a location constant in time.

On the other hand, the bid auction approach can handle the effect of changes in the market conditions because prices are a function of the preferences of the households, and bids can be adjusted to react against an increase or decrease of supply/demand. However, this approach has only been implemented in aggregated, equilibrium based, models.

This paper proposes a method to model location choice and real estate prices simultaneously in a microsimulation context. The method is based on the bid-auction approach and estimates both location and prices as a function of the households' preferences. The proposed approach does not require solving for equilibrium, but estimates the maximum bid in each period by simulating the underlying auction process. Households adjust their preferences (and their willingness to pay) as a reaction to the (observed) market conditions. Demand surplus triggers a more competitive market, therefore increasing the willingness to pay, while supply surplus have the opposite effect.

# 2 The bid approach

Since Alonso (1964), the real estate market has been understood as an auction market, where households bid their willingness to pay for a particular good (residential unit) which is assigned to the best bidder. This process simultaneously defines the price of the good, understood as the maximum bid in the auction process.

The willingness to pay, from an economic point of view, can be derived from the classical consumer's problem of maximum utility, given income constraints:

$$\max_{x,i} U(x, z_i) \tag{1}$$
$$s.t.\ px + r_i \leq I$$

In the previous problem, the consumer maximizes his utility by choosing a vector of continuous goods ($x$) and a discrete location ($i$), described by a set of attributes ($z_i$). The budget constraint states that the total amount spent in goods (with price $p$) plus the price of the selected location ($r_i$) must be smaller that the consumer's available income ($I$). Solving the problem on $x$ and assuming equality in the budget constraint, the problem can be re-written as

$$\max_{i} V(p, I - r_i, z_i) \tag{2}$$

where $V$ is the indirect utility function, conditional on the the location. Conditional on the level of maximum utility ($\overline{U}$), the indirect utility can be inverted in the rent variable:

$$r_i = I - V^{-1}(\overline{U}, p, z_i) \tag{3}$$

Under the auction market assumption, the rent variable can be understood as the willingness to pay for a particular location, therefore the bid function $B$ can be expressed as:

$$B_{hi} = I_h - V_h^{-1}(\overline{U}, p, z_i) \tag{4}$$

In the bid function, the index $h$ has been included to take into account heterogeneity in preferences within different households. If we assume bids to be random variables, with an extreme value distributed error term, it is possible to express the probability of a household ($h$) being

the best bidder for a particular location ($i$) as follows:

$$P_{h/i} = \frac{\exp(\mu B_{hi})}{\sum_g \exp(\mu B_{gi})} \tag{5}$$

Under the auction market assumption, the price or rent of a good will be the maximum bid. The extreme value distribution assumption allows to write the expected maximum bid for a particular location as the logsum of the bids

$$r_i = \frac{1}{\mu} \ln \left( \sum_g \exp(\mu B_{gi}) \right) \tag{6}$$

Under the bid approach, rents can only be determined when rents have been adjusted to ensure that each household is located somewhere and in not more than one location. This means that the utility level of each household should be adjusted to ensure that:

$$\sum_i P_{h/i} = 1 \tag{7}$$

The previous condition is only possible when and absolute equality between supply (the number of location alternatives) and demand (the number of households) holds, meaning that:

$$\sum_h \sum_i P_{h/i} = H = S \tag{8}$$

with $H$ the total number of households and $S$ the total number of locations.

# 3 The choice approach

The choice approach (McFadden, 1978; Anas, 1982) assumes that households choose the location that maximize their utility. The utility a household perceives is the indirect utility function (2) and can be defined as a function of the attributes of the location ($V_{hi} = f(z_i)$). Assuming an extreme value distribution for the error term of the utility function, the probability of a household $h$ choosing a location $i$ is:

$$P_{i/h} = \frac{\exp(\mu V_{hi})}{\sum_j \exp(\mu V_{hj})} \tag{9}$$

It is possible to demonstrate that, under the assumption of an auction market, the location where the agent is the highest bidder is also that of the maximum surplus or maximum utility (Martinez, 1992, 2000). This assures that the auction outcome yields an allocation consistent with maximum utility behavior of consumers. The consumer surplus is defined as the difference between the willingness to pay for a good and the actual price of the good. If the utility is written in terms of consumer surplus it will take the following form:

$$V_{hi} = B_{hi} - r_i \tag{10}$$

Replacing (10) in (9), the probability of a household $h$ choosing a location $i$ is:

$$P_{i/h} = \frac{\exp(\mu(B_{hi} - r_i))}{\sum_j \exp(\mu(B_{hj} - r_j))} \tag{11}$$

If prices are the outcome of an auction process and the market clears, the distribution of households across locations obtained through (11) will be the same as the distribution obtained from (5).

# 4 The relation between bid rent and hedonic rent

Following Rosen (1974)'s approach, real estate prices or rents can be expressed as a function of the attributes of the location ($r_i = f(z_i)$). In fact, most of the operational land use microsimulation models, like UrbanSim (Waddell, 2002), use hedonic prices in their formulations. The most common form for a hedonic price model is a linear in parameters function:

$$r_i = \sum_k \alpha_k z_{ik} \tag{12}$$

where $k$ is an index for the $k^{th}$ attribute of the location. The parameters in a hedonic prices model can be interpreted as the market value of each of the attributes:

$$\alpha_k = \frac{\partial r_i}{\partial z_{ik}} \tag{13}$$

Under the assumption of an auction market (bid approach), the market value for each of the attributes (that is, the price at which this attribute would be bought) can be expressed as the derivative of the logsum (equation 6) with respect to the attribute. Since the attributes appear

in the bid function of each household, the derivative takes the following form

$$\frac{\partial r_i}{\partial z_{ik}} = \sum_h \left( \frac{\partial \left( \ln \left( \sum_g \exp(B_{gi}) \right) \right)}{\partial B_{hi}} \cdot \frac{\partial B_{hi}}{\partial z_{ik}} \right) \tag{14}$$

If the bid function is also linear in parameters ($B_{hi} = \sum_k \beta_{hk} z_{ik}$) we have:

$$\frac{\partial r_i}{\partial z_{ik}} = \sum_h \left( P_{h/i} \cdot \beta_{hk} \right) \tag{15}$$

Therefore, if the prices are the outcome of an auction, the standard hedonic model will be an approximation of the maximum expected bid, where the parameter $\alpha$ tries to reproduce a weighted average of the individual households preferences $\beta$. However, it's hard to reproduce the maximum expected bids using hedonic models because they do not take into account the adjustment in the willingness to pay of each household (Hurtubia *et al.*, 2010).

The result of (15) allows to understand the bid function as having a hedonic component, function of the attributes of the location and the consumers' preferences. Therefore, assuming that land is sold in auctions, a direct mapping between consumers utility functions and the corresponding hedonic rent functions can be assumed.

# 5 Bid rent model for microsimulation

Microsimulation of land use requires a representation at the individual level of the location choice and price formation processes. This means that each household is paired to a location in a sequential way

The choice approach is straightforward to implement in a microsimulation context because it provides the individual location probabilities and rents are calculated exogenously (and independently) for each dwelling following an hedonic model without requiring any assumption about equilibrium between supply and demand. However, implementing the choice approach requires the assumption that supply will always satisfy demand, so the allocation process can be simulated by drawing a location for each household. The order in which the allocation happens can only be assumed to be random, drawing the location for each household at a time and making selected location unavailable for future choices. If a demand surplus scenario happens, a choice approach will only be able to deal with this by randomly selecting households that will not be located.

Implementing a bid approach is not straight forward, because prices can only be determined if a supply-demand equilibrium is achieved and bids are adjusted to this. The complexity comes from the fact that equality between demand and supply is usually not guaranteed in a microsimulation (because of an independent supply generation process). Also, the bid approach traditionally assumes that each location "chooses" a household through the auction process, therefore making hard to simulate scenarios with supply surplus (there is no clear rule to decide which locations are not used).

We propose a model where, at each period of time, the auction for each good is simulated, therefore obtaining rent levels that reflect the competition between different bidders for the good. The adjustment accounts for the effect a supply or a demand surplus will have on the bids. We solve the allocation problem by proposing a different market clearing solution depending on the supply/demand surplus conditions of the scenario

We assume the bid function to be composed of two elements, therefore, for a particular period $t$:

$$B_{hi}^t = b_h^t + b_{hi}^t(z_i)$$

where $b_h^t$ is the adjustment component that relates the bid with the utility level of the household and $b_{hi}^t$ is the hedonic part of the bid expressing the value a household $h$ gives to the attributes $(z_i)$ of a location $i$. We assume the preferences of households remain constant in time, therefore the value of the hedonic part of a bid for a particular pair $(h, i)$ will remain constant unless the attributes of the location change. The market conditions change from one period to the other (population , income levels, available supply, etc.) and the term $b_h$ reacts to these changes, therefore having different values in each period.

The adjustment of $b_h$ follows the logic of households increasing or decreasing their bids depending on the conditions of the auction (or, in more general terms, the market). In each auction, if there is a demand surplus households will try to outbid other households until reaching an expected average outcome of winning auctions that allows to locate "somewhere" (although it does not ensure their location). Similarly, in the presence of supply surplus, households will reduce the level of their bids because they can reach an expected number of winning auction that allows to locate somewhere with smaller bids

In each period, the knowledge of the state of the market comes from the observed rents from previous periods $(r_i^t)$. We assume that households also observe the available supply $(S^t)$ and know the number of households looking for a location in each period $(H^t)$. However, we assume they don't observe the bids of other households (therefore our system represents a sealed-bid auction). Considering this information each household estimates the value of $b_h^t$

required to make the expected number of winning auctions equal to one.

$$\sum_i P^t_{h/i} = \sum_{i \in S^t} \frac{\exp\left(\mu(b^t_h + b^t_{hi}(z_i))\right)}{\sum_{g \in H^t} \exp(\mu B^t_{gi})} = 1 \tag{16}$$

Since households can't observe the bids of other households in $t$ we assume they observe the bids in the previous period $(t-1)$. This is equivalent as observing the rents in the previous period since, following 6, the denominator of (16) can also be expressed as:

$$\sum_{g \in H} \exp(\mu B^t_{gi}) = \exp(\mu r^t_i)$$

Clearing $b^t_h$ from (16) we obtain:

$$b^t_h = -\ln\left(\sum_{i \in S^t} \exp \mu \left(b^t_{hi}(z_i) - r^{t-1}_i\right)\right) \tag{17}$$

After the adjustment of $b_h$is calculated it's possible to calculate the real probabilities and rents in $t$.

## 5.1 Allocation process

We assume the allocation process will happen in two different ways, depending on the general conditions of the market regarding (demand or supply) surplus. The number of located households or used dwellings may differ from the total number of active households or locations in the market. We denote the set of located households in a period as $\hat{H}^t$ and the set of used locations in the same period as $S^t$.

In a period with demand surplus it is impossible to allocate all households because of the insufficient demand. As explained before, households will increase the level of the bid as a reaction to this. However, some of the households will be outbid in every auction and remain unlocated. The market conditions make then more appropriate to use the bid probabilities ($P_{h/i}$) to simulate the allocation of households to dwellings. It makes sense to do this location wise, following (5), as if each location was selecting the best bidder from the pool of remaining households

In the opposite case, in a period with supply surplus, not all the dwelling will be used. Therefore, a choice probability ($P_{i/h}$) seems more appropriate to simulate the allocation of dwellings

to households, following (11).

In any market, the transactions are usually bounded by structural characteristics of the involved agents. In the case of the real estate market the constrains are given by the maximum feasible bid for each household (usually determined by the income level) and the reservation price (or minimum feasible rent) of each location. For simplicity, these constraints are ignored in the current formulation of the model, meaning that prices can go has high or low as required by the adjustment of (18). This means that, in the case of demand surplus, all dwellings ($S^t$) will be used while only a fraction of the total households will be located. Similarly, in the case of a supply surplus scenario, all households ($H^t$) are expected to be located while only a fraction of the dwellings will be used.

Introducing a constrained behavior in the bidding/selling process requires to define thresholds which trigger the exclusion of a household or a dwelling from the transaction. This would allow the existence of (more realistic) scenarios where some households are not located while, at the same time, some dwellings are not occupied. The inclusion of the thresholds should generate a non-compensatory location probability, which can be modeled using models like the Constrained Multinomial Logit (Martinez *et al.*, 2009). An example of the use of non-compensatory probabilities for location choice, but in the context of equilibrium models, can be found in Martinez and Hurtubia (2006)

# 6 Estimation

Implementation of the proposed model requires to estimate the parameters of the bid function for a base period. This means that both the utility level ($b_h^0$) and the hedonic part ($b_{hi}^0$) of the bid function should be estimated as if a full equilibrium was taking place between the observed households and dwellings at the base period. The process then requires to first estimate the parameters of the hedonic part through maximum log likelihood following (5) and, second, to adjust the value of $b_h^0$ following

$$b_h^0 = -\ln\left(\sum_{i\in S^0} \exp\mu\left(b_{hi}^0(z_i) - r_i^0\right)\right) \tag{18}$$

The solution of the previous equation implies a fixed point problem because the rents ($r_i^0$) depend on $b_h^0$, as defined by equation (6). It is important to know the total number of households and available dwellings at the base period in order to introduce the effect of the structural vacancy rate in the rent.

Once the equilibrium bids and rents have been obtained, they are used as the input for the

Table 1: Location in base period

| zone | poor hh ($P$) | rich hh ($R$) | total supply | rent ($r_i$) |
|---|---|---|---|---|
| 1 ($z_1 = 0.5$) | 281 | 219 | 500 | 1.25 |
| 2 ($z_1 = 1.0$) | 219 | 281 | 500 | 2.00 |
| total demand | 500 | 500 | 1000 | |

simulation of the first period of the simulation.

# 7 Simplified experiment

A simple experiment is conducted to test the properties of the proposed models regarding proper reaction to changes in the market conditions. For this a vary simple synthetic city is built considering only two possible zones for location and only two types of households. The zones have only one attribute that characterize them, having a zone with a low value ($z_1 = 0.5$) and a zone with a high value ($z_2 = 1.0$) of the attribute. Households show either a high willingness to pay for the attribute (rich households, with $\beta_R = 2$) or a low willingness to pay for the attribute (poor households, with $\beta_P = 1$). For simplicity, and in order to allow a better analysis of the reaction to general market conditions of the model, the attributes of the zones remain constant in time. This can be interpreted as not accounting for location externalities in the model.

In the base period the city is perfectly equilibrated, with 500 dwellings in each zone and 500 households of each type. Table 1 shows the location and rents after the equilibrium.

The simulations are done for 20 periods after the base one. Two different scenarios are simulated: one with a supply surplus and one showing demand surplus.

## 7.1 Supply surplus scenario

In the supply surplus scenario real estate developers predict accurately the total future demand for every period but the first one, where an (arbitrary) overproduction of dwellings take place. Demand grows homogeneously while supply slowly adjusts to match it. Figure 1 show the resulting rents when applying the proposed model. As expected, rents are higher for dwellings in the zone with higher values for the attributes. In the first period, the excess of supply triggers a reduction in the rents thats continues for several periods until supply matches demand again (around period 5). After this point, and given the equality between supply and demand, rents increase until they reach the original (equilibrium) levels.

Figure 1: Rents for the supply surplus scenario

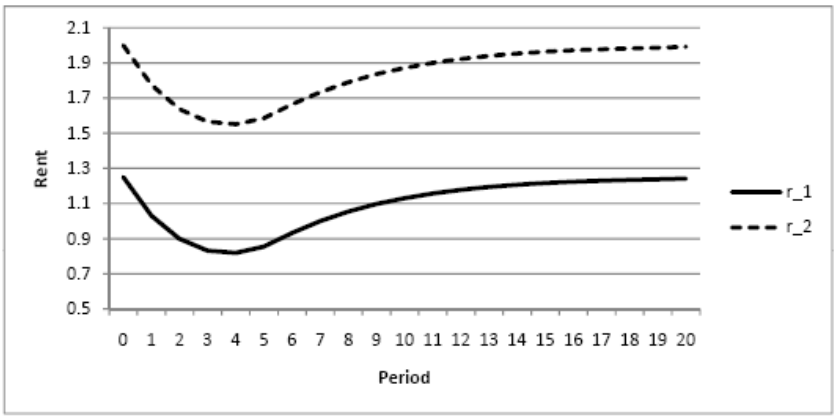
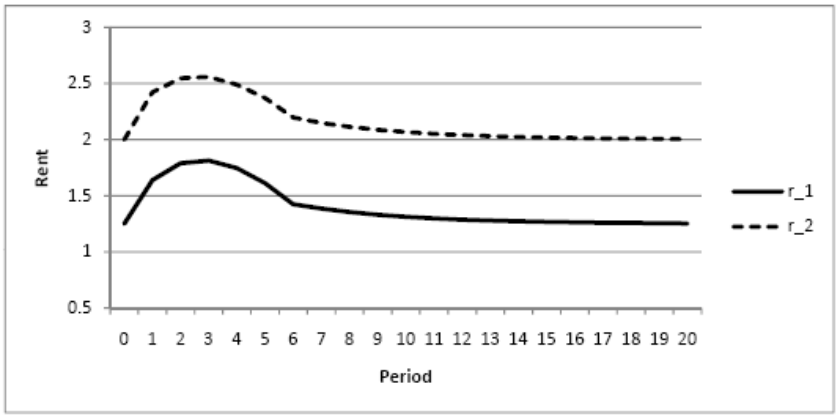



Figure 2: Rents for the demand surplus scenario




## 7.2 Demand surplus scenario

The demand surplus scenarios is generated by producing a shock in the growth for rich households in the first period. Supply is unable to react inmediatly to this and does so in a slow manner.

Figure 2 shows the rents for this scenario. The excess of demand generates an increase in the rent which decreases slowly as supply approaches the levels required to satisfy demand. After several periods rents return to the original equilibrium levels

# 8 Conclusions

The proposed model is able to account for the auctioning process that takes in each period of a simulation. The advantage of the model lies in the fact that is able to account for changes in the general conditions of the market, like a growth or a reduction of the ratio between demand and available supply. The method is in based in a bid approach for location choice modeling. How-

ever it simulates the location process of individuals as the outcome of a bid (dwellings selecting the best bidder/household) only when a demand surplus situation is observed. In the case of a demand surplus scenario, the model simulates the location as a choice (households selecting the location that maximizes their utility). Future work will consist in the implementation of a simulation accounting for location externalities and increasing the heterogeneity in both supply and demand agents. Application to real data and validation will be done in the context of the SustainCity project (www.sustaincity.org), specifically to the city of Brussels.

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