Rebalancing Idle Vehicles via Distributed Coverage Control in Mobility-on-Demand systems

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Idle/empty vehicles have no destinations. Where should they go?
Imbalance in the spatial distribution of vehicles:

- Non-uniform passenger’s demand for rides in different districts
- Asymmetry between origin and destination distributions of trips

Goal: rebalancing vehicles

- Relocating idle/empty vehicles to the high-demand regions
Vehicle rebalancing problem

Coverage control problem:

Every agent/vehicle is responsible for covering a certain area

$$H(X, W) = \sum_{i=1}^{n} H(x_i, W_i') = \sum_{i=1}^{n} \int_{q \in W_i'} f(||x_i - q||^2) \varphi(q) dq$$

$$\varphi(q)$$: demand density function,
$$f: [0, \infty) \rightarrow \mathbb{R}$$, a performance function which degrades with distance.

Ref: S. Martinez(2007), J. Cortes and F. Bullo, "Motion Coordination with Distributed Information"
Voronoi partition:

- The partitioning of a plane with \( n \) points into convex polygons
- Each cell contains one generator/seed
- Every point in a given cell is closer to its seed than to any other

- With Voronoi diagram, we can disperse the vehicles in the region
Coverage Control Algorithm

Important variables:
- \( N \) = Number of agents
- \( r \) = Agent coverage radius

Objective:
Maximize the value of \( H \)

\[
H(X,W) = \sum_{i=1}^{n} H(x_i, W_i) = \sum_{i=1}^{n} \int_{q \in W_i} f(\|x_i - q\|^2) \varphi(q) dq \\
W_i = S_i \cap V_i
\]
Distributed Coverage Control Algorithm

Important variables:
- $N$ = Number of agents
- $r$ = Agent coverage radius
- $R$ = Agent communication limitation radius

Objective:
Maximize the value of $H$

$$H(X,W') = \sum_{i=1}^{n} H(x_i, W_i') = \sum_{i=1}^{n} \int_{q \in W'_i} f(||x_i - q||^2)\varphi(q)dq$$

$$W_i' = S_i \cap V_i'$$
Proposition: The local maximum of $H$ can be obtained when all $x_i$ are located at centroids (centers of mass, $C_{W'_i}$) of their respective Voronoi cells ($W'_i$), i.e., *Centroidal Voronoi Configuration* (CVC).

**Distributed Control Law Formulation:**

$$\frac{dx_i}{dt} = u_i, \quad \frac{\partial H}{\partial x_i} = -2M_{W'_i} \| x_i - C_{W'_i} \|,$$

$$u_i = -k_p (x_i - C_{W'_i}),$$

$$\frac{\partial H}{\partial t} = \sum_{i=1}^{n} \frac{\partial H}{\partial x_i} \frac{dx_i}{dt} = 2k_p \sum_{i=1}^{n} M_{W'_i} \| x_i - C_{W'_i} \|^2 > 0.$$

which steer the agent team to converge to CVC.

Case Study I: Continuous Case

(a) Demand density function
(b) Initial configuration
(c) Final configuration
Case Study II: Shenzhen, China

Simulated network:
- Luohu District of Shenzhen, China
- 1858 nodes
- 2013 links
- 199,819 trips consisting of origins, destinations, and time

Experimental Setup

- 3-hour simulation
- Time Pattern of demand: low-high-low, each period lasts for 1 hour
- 2400 orders
- Fleet size = 100

- Private vehicles; Ride-hailing vehicles
- macroscopic fundamental diagram (MFD) for Shenzhen

\[ v(n) = \begin{cases} 
\frac{29m}{36e^{\frac{m}{600}}}, & \text{if } m \leq 36, \\
6.31 - 0.28(m - 36), & \text{if } 36 < m \leq 60, \\
0, & \text{if } m > 60 
\end{cases} \]

where \( m = \frac{n}{1000} \).
blue: idle vehicle (i.e., empty, looking for a passenger),
red: passenger-assigned vehicle,
green: passenger-carrying vehicle.
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red: passenger-assigned vehicle,
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Performance metrics

- Order completion rate:
  \[ \frac{N_1}{N} = \frac{N_1}{N_1 + N_2} \times 100\% \]

- Average Waiting time:
  \[ t_w = \frac{\sum_{i=1}^{N_1} (t_p^r(i) - t_o(i))}{N_1} \]

- Average System time (with penalty for cancelled orders):
  \[ t_{sys} = \frac{\sum_{i=1}^{N_1} (t_p^r(i) - t_o(i)) + N_2 \cdot \alpha \cdot w_{tol}}{N} \]
  \[ \alpha = 1.5, w_{tol} = 5 \text{ min} \]
• 3-hour simulation
• Time Pattern of demand: low-high-low, each period lasts for 1 hour
• 2400 orders
• Fleet size = 150

<table>
<thead>
<tr>
<th></th>
<th>Proposed method</th>
<th>Do-nothing policy</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completion rate(%)</td>
<td>82.9</td>
<td>73.2</td>
<td>13.3%↑</td>
</tr>
<tr>
<td>Average waiting time(s)</td>
<td>132.5</td>
<td>173.9</td>
<td>23.8%↓</td>
</tr>
<tr>
<td>Average system time(s)</td>
<td>186.8</td>
<td>247.9</td>
<td>24.6%↓</td>
</tr>
</tbody>
</table>
- Operate the fleet more efficiently as a larger amount of vehicles are actively serving passengers.
\( p'_d(q) = \gamma \cdot p_d(q) + (1 - \gamma) \cdot p'_\delta(q) \)

where \( p'_\delta \) is an artificial distribution which has the maximum difference from the origin distribution.

- When \( \gamma = 1 \), the generated destination distribution is equal to the original destination distribution.
- The smaller the \( \gamma \) is, the more discrepancy is introduced between the origin and generated destination distributions.
- When \( \gamma = 0 \), the generated destination distribution has a shape that is maximally different than the origin one.
Comparison of performance metrics for various $\gamma$ and fleet size value

- Fleet size
- Gamma

(a) Results with varying values of origin destination demand imbalance parameter $\gamma$.

(b) Results with varying values of fleet size.
Comparison of performance metrics for various $\gamma$ and fleet size value

(a) Results with varying values of origin destination demand imbalance parameter $\gamma$.

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- Fleet size
- Gamma
Comparison of performance metrics for various $\gamma$ and fleet size value

- Fleet size

(a) Results with varying values of origin destination demand imbalance parameter $\gamma$.

- Gamma

(b) Results with varying values of fleet size.

Average system time
Comparison of performance metrics for various \( r \) and \( R \)

(a) Results with varying values of covering radius \( r \).

(b) Results with varying values of the communication limitation radius \( R \).
Conclusion

Demand density function + Control Algorithm

Local information

Relocate position
Conclusion

Distributed Coverage Control Algorithm:

- Application to rebalancing of vehicle fleets for urban Mobility-on-Demand systems

- Countering spatiotemporal imbalances in the origins and destinations of trip demands

- Dynamically rebalance spatial distribution, serve more trips with less waiting time.

- Tested on both continuous and discrete space compared with a do-nothing policy.

- The effects of coverage and communication radius are demonstrated respectively.
Future steps

1. Estimate time-varying demands from real-time data
2. Ride-sharing/pooling
3. Detour problem
4. Price Policy
Thanks for your attention!

Avez-vous des questions?

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