Nonlinear model predictive variable speed limit control of freeway systems

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Swiss Federal Institute of Technology in Lausanne **May 2019**

STRC | 19th Swiss Transport Research Conference
Monte Verità / Ascona, May 15 – 17, 2019

Swiss Federal Institute of Technology in Lausanne

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May 2019

Abstract

Traffic conditions in freeways can be improved using congestion control systems with actuation over variable speed limits (VSLs). Considering the availability of high accuracy dynamical models such as the cell transmission model, and the presence of physical constraints on VSLs and possibly on freeway densities, nonlinear model predictive control (MPC) emerges as a strong candidate for the design of high performance freeway congestion management systems. In this paper we propose a nonlinear MPC scheme that can steer the freeway system to a density equilibrium that maximizes throughput. Performance of the proposed scheme is demonstrated through simulations.

Keywords

Freeway congestion management; cell transmission model; variable speed limit; nonlinear model predictive control.

1 Introduction

Freeway traffic flow management is becoming critical due to increasing levels of congestion. Intelligent transportation systems enable efficient operation of existing freeway infrastructure, by making proper use of the limited road space through various actuation systems such as ramp metering, route guidance, and variable speed limits. In particular, combining modern traffic flow modeling techniques with advanced feedback control methods opens up many possibilities for the design of modern freeway traffic control systems with high reliability and performance.

Variable speed limit (VSL) is emerging as an actuation technique that is receiving increasing interest in the freeway traffic control literature. In VSL the idea is to install, at specific locations on freeway stretches, speed limit signs that can be changed in real time for responding to prevailing traffic conditions. It is possible to improve mobility by VSL through manipulating speed limits upstream of a freeway mobility (Kotsialos *et al.,* 2002, Hegyi *et al.,* 2008, Carlson *et al.,* 2013, Frejo *et al.,* 2014, Zhang and Ioannou, 2016).

Many works in recent literature focus on feedback control design for VSL-actuated freeway systems. A local feedback control method is proposed in Carlson *et al.* (2013), which is extended to the case of multiple bottlenecks in Iordanidou *et al.* (2015), and to the case of upstream delay balancing in Iordanidou *et al.* (2017). The problem of maximizing bottleneck throughput is treated with a proportional-integral VSL controller in Jin and Jin (2015), which can locally stabilize density at its critical value using one VSL actuator upstream of the bottleneck. A lane change (LC) control method is combined with a feedback linearization (FL) approach in Zhang and Ioannou (2016), that is able to deal with freeway traffic management under incident conditions involving a blocked lane.

Model predictive control (MPC) has seen recent interest in freeway control literature. MPC is an advanced control technique involving repeated optimal control which relies on dynamic optimization problems solved in real time. In MPC, at each time step during real-time operation, a finite horizon constrained optimal control problem (constructed using a mathematical model of the system to be controlled) is solved based on current measurements, and the first element of the optimal control inputs vector is applied to the system. The whole procedure is repeated at the next time step, creating a feedback control mechanism.

A number of recent works focused in particular on VSL-actuated MPC approaches for freeway systems. Employing a car-following model, an MPC approach is presented in Zegeye *et al.* (2009) that is able to reduce congestion and total emissions. In Frejo *et al.* (2014), a hybrid

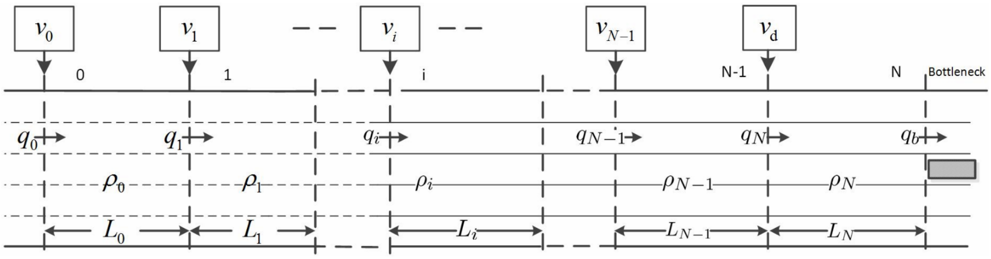

Figure 1: Schematic depiction of VSL-actuated freeway system.

MPC combining ramp metering and VSL type actuators is proposed. An MPC design based on the link-node cell transmission model is given in Muralidharan and Horowitz (2015). In Han *et al.* (2017) an MPC is developed based on a discrete first order model capturing jam wave propagation. In this paper we develop a nonlinear MPC scheme for VSL-actuated freeway systems under incident conditions.

2 Modeling

We base this section on the setting and approach taken in Zhang and Ioannou (2016). Consider a VSL-actuated freeway system (see Fig. 1) consisting of $N+1$ sections with possibly different lengths, with a blocked lane at the location just downstream of section N causing a bottleneck. The vehicle mass conservation equations describing the the freeway system dynamics can be written as follows:

$$\begin{aligned} \dot{\rho}_i(t) &= \frac{q_i(t) - q_{i+1}(t)}{L_i} \quad \text{for } i = 0, 1, ..., N-1 \\ \dot{\rho}_N(t) &= \frac{q_N(t) - q_b(t)}{L_N}, \end{aligned} \tag{1}$$

where ρ_i (veh/mi), v_i (mph), q_i (veh/h), and L_i (mi) are the vehicle density, VSL control input, received vehicle flow, and length, for section i, respectively. The VSL control input is fixed to a constant value v_d for section N, while the bottleneck flow is denoted q_b. Vehicle flows q_i between the sections can be modeled using the cell transmission model (Daganzo, 1994) and assuming triangular fundamental diagrams:

$$\begin{aligned} q_0(t) &= \min\{d, C_0, w_0(\rho_{j,0} - \rho_0(t))\} \\ q_i(t) &= \min\{v_{i-1}(t)\rho_{i-1}(t), C_i, w_i(\rho_{j,i} - \rho_i(t))\} \quad \text{for } i = 1, 2, ..., N, \end{aligned} \tag{2}$$

where d (veh/h) is the constant demand flow, $\rho_{j,i}$ (veh/mi) is the jam density of section i, w_i (mph) is the backward propagating wave speed of section i, and C_i (veh/h) is the capacity (maximum possible flow) of section i.

During incident situations (as depicted in Fig. 1 with the blocked middle lane downstream of section N), the section upstream of the bottleneck experiences capacity drop. Arising due to irregularities in traffic flow caused by forced lane changes in the vicinity of the incident, capacity drop causes discontinuity in the fundamental diagram. Employing a lane changing controller such as the one developed in Zhang and Ioannou (2016) it is possible to regularize traffic flows by providing early warning through lane change recommendations to drivers starting from a certain distance (e.g., around 800 m for a freeway system with 5 lanes with 1 blocked lane under medium demand conditions) upstream of the incident. Deployment of this lane changing control mechanism can eliminate capacity drop and thus a discontinuity in the bottleneck fundamental diagram, which enables the use of a continuous model instead:

$$q_{\mathrm{b}}(t) = \begin{cases} v_{\mathrm{d}}\rho_N(t) & \text{if } \rho_N(t) \le \rho_{\mathrm{d,c}} \\ w_{\mathrm{b}}(\rho_{\mathrm{j,d}} - \rho_N(t)) & \text{otherwise,} \end{cases} \tag{3}$$

where w_{b} (mph), $\rho_{\mathrm{d,c}}$ (veh/mi), and $\rho_{\mathrm{j,d}}$ (veh/mi) are the backward propagating wave speed, critical density, and jam density, respectively, for the bottleneck section.

Assuming that the sections upstream of the bottleneck have capacities larger than the bottleneck capacity (which is the usual case in reality), the system dynamics Eq. (1) can be rewritten as:

$$\begin{aligned} \dot{\rho}_0(t) &= \frac{w_0(\rho_{\mathrm{j},0} - \rho_0(t)) - v_0(t)\rho_0(t)}{L_0} \\ \dot{\rho}_i(t) &= \frac{v_{i-1}(t)\rho_{i-1}(t) - v_i(t)\rho_i(t)}{L_i} \quad \text{for } i = 1, 2, \ldots, N-1 \\ \dot{\rho}_N(t) &= \frac{v_{N-1}(t)\rho_{N-1}(t) - q_{\mathrm{b}}(t)}{L_N}. \end{aligned} \tag{4}$$

The system dynamics Eq. (4) is non-differentiable due to the triangular fundamental diagram expressing the bottleneck flow $q_{\mathrm{b}}(t)$. Since nonlinear MPC requires smooth nonlinear dynamics, we approximate the bottleneck fundamental diagram with a sixth degree polynomial:

$$\tilde{q}_{\mathrm{b}}(t) = p_1\rho_N^6(t) + p_2\rho_N^5(t) + \ldots + p_6\rho_N(t), \tag{5}$$

where p_k ($k = 1, \ldots, 6$) are the polynomial coefficients which can be found by fitting Eq. (5) to the triangular fundamental diagram Eq. (3).

3 Control Design

We formulate the problem of finding the VSL control inputs $v(\cdot)$ that try to steer the freeway system to equilibrium as the following nonlinear MPC problem:

$$\begin{aligned}
\underset{v(\cdot)}{\text{minimize}} \quad & \int_0^{T_p} e_\rho(\tau)^T Q e_\rho(\tau) + e_v(\tau)^T R e_v(\tau) d\tau \\
\text{subject to} \quad & \rho(0) = \hat{\rho}(t) \\
& \forall \tau \in [0, T_p] : \\
& \quad \dot{\rho}(\tau) = f(\rho(\tau), v(\tau)) \\
& \quad v_{\min} \leq v(\tau) \leq v_{\max},
\end{aligned} \tag{6}$$

where T_p is the prediction horizon, τ is the time internal to the MPC, Q and R are the weighting matrices for the density and VSL input error terms, respectively, $\hat{\rho}(t)$ is the density measurement taken at real time t, $f(\cdot)$ is the function representing the dynamics Eq. (4) with Eq. (5), $\rho(\cdot)$ and $v(\cdot)$ are vectors containing all density ρ_i ($i = 0, 1, \ldots, N$) and VSL input terms v_i ($i = 0, 1, \ldots, N-1$), respectively, $v_{\min}$ and $v_{\max}$ are bounds on the VSL inputs, while $e_\rho(\cdot)$ and $e_v(\cdot)$ are density and VSL input error terms, respectively, expressing the deviations from the equilibrium point:

$$e_\rho = \rho - \rho_e \qquad e_v = v - v_e, \tag{7}$$

where ρ_e and v_e denote the density and VSL input vectors at the equilibrium.

Equilibrium point should maximize the bottleneck flow $q_{\mathrm{b}}(t)$, and homogenize the densities and speeds over the whole system. The density that maximizes $q_{\mathrm{b}}(t)$ is the critical density, and for homogenizing the densities and speeds, we choose for sections 1 to N:

$$\rho_{e,i} = \rho_{\mathrm{d,c}} \qquad v_{e,i} = v_{\mathrm{d}} \quad \text{for } i = 1, \ldots, N, \tag{8}$$

while for section 0, since the demand d is larger than the bottleneck capacity C_b, the flow entering the freeway system needs to suppressed so as to make section 0 operate at bottleneck capacity C_b:

$$v_{e,0}\rho_{e,0} = w_0(\rho_{\mathrm{j},0} - \rho_{e,0}) = C_b, \tag{9}$$

which yields the equilibrium for section 0:

$$\rho_{e,0} = \rho_{\mathrm{j},0} - \frac{C_b}{w_0} \qquad v_{e,0} = \frac{C_b}{\rho_{e,0}}. \tag{10}$$

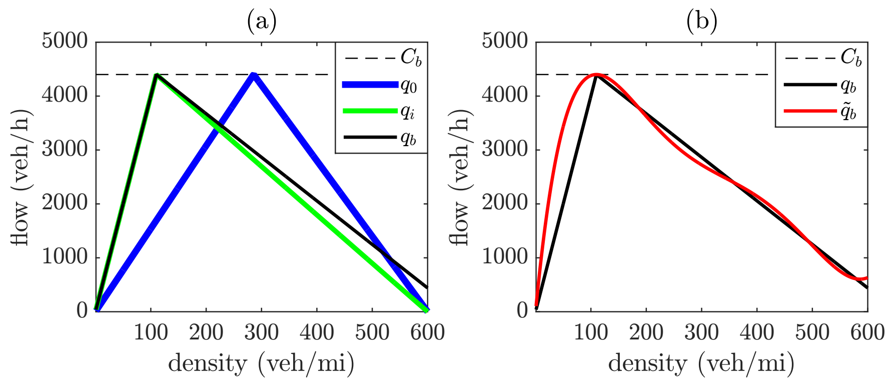

Figure 2: Fundamental diagrams (a) q_0, q_i ($i = 1, \ldots, N$), and q_b, and (b) q_b and its polynomial approximation $\tilde{q}_b$, together with bottleneck capacity C_b.

4 Results

The proposed nonlinear MPC method is tested on a 10 section freeway system under incident conditions, which is represented in simulation by the dynamical model given in Eq. (1) under a constant demand of $d = 6000$ veh/h and a bottleneck capacity of $C_b = 4400$ veh/h. The bottleneck critical density and equilibrium speed are $\rho_{\mathrm{d,c}} = 110$ veh/mi and $v_{\mathrm{d}} = 40$ mph, respectively. Jam density of all sections is $\rho_{\mathrm{j},i} = 600$ veh/mi ($i = 0, \ldots, N$), while the bottleneck jam density is $\rho_{\mathrm{j,d}} = 654$ veh/mi. Backward propagating wave speed is $w_0 = 14$ mph for section 0, $w_i = 9$ mph for sections 1 to N, while it is $w_b = 9$ mph for the bottleneck. Equilibrium density and speed of section 0 is $\rho_{e,0} = 285.7$ veh/mi and $v_{e,0} = 15.4$ mph, respectively. Fundamental diagrams corresponding to these parameter configuration are given in Fig. 2.

The nonlinear MPC scheme is built using direct multiple shooting (Bock and Plitt, 1984), with dynamics discretized via Runge?Kutta method with a timestep of 30 seconds. The implementation is done using MPCTools (Risbeck and Rawlings, 2016) (an interface to CasADi (Andersson *et al.*, 2018)) with IPOPT (Wächter and Biegler, 2006) as solver. Prediction horizon of the nonlinear MPC is chosen as 25, while the weighting matrices are $Q = I_{N+1}$ and $R = 0.1 I_N$. VSL control inputs are bounded as 10 mph $\leq v_i(t) \leq$ 65 mph. Time step of the simulation model is 30 seconds, while the total simulation length is 1 hour.

The simulation results are depicted in Fig. 3, which contains the density and VSL control input trajectories of all sections for the total simulation length. In the figure a comparison between the proposed nonlinear MPC scheme with a no control case is presented, where the no control case involves fixing the VSL inputs at their equilibrium values. The results show that using nonlinear MPC it is possible to steer the freeway system quickly to the desired equilibrium point, indicating its potential for the design of high performance freeway control system design.

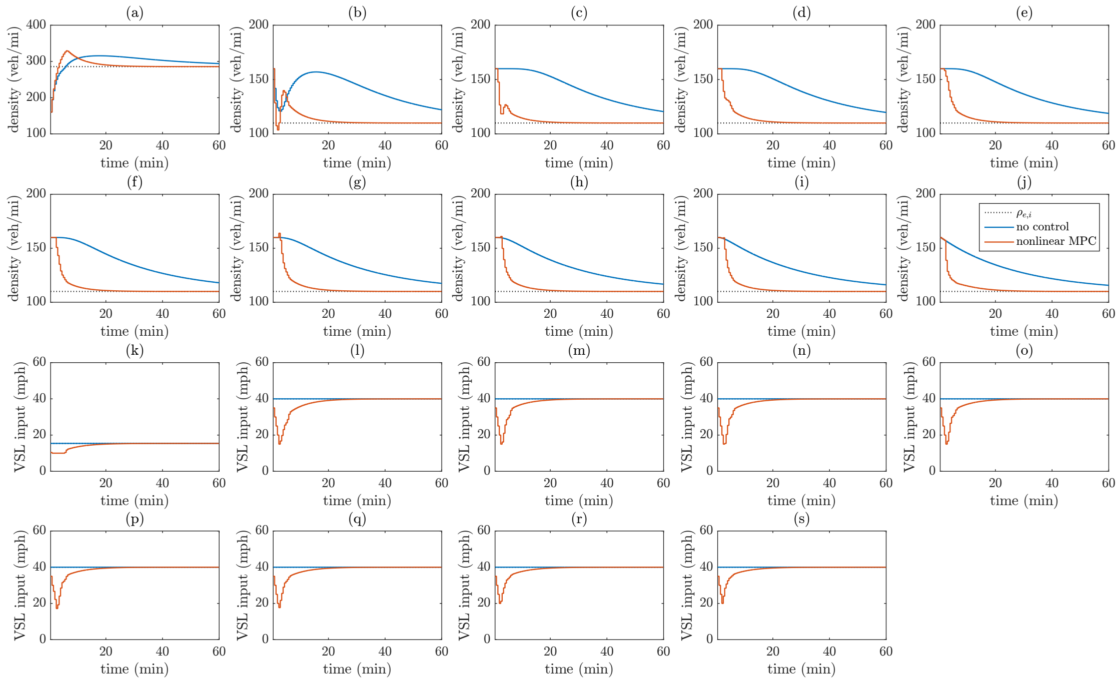

Figure 3: Simulation results depicting (a)-(j) densities $\rho_i(t)$ $(i = 0, \dots, 9)$ and (k)-(s) VSL control inputs $v_i(t)$ $(i = 0, \dots, 8)$, comparing the nonlinear MPC (red) with no control (blue).

5 Conclusion

In this paper we developed a nonlinear MPC scheme for freeway systems under incident conditions with dynamics described by cell transmission model. Simulation results indicate that the proposed scheme can steer the freeway system quickly to an equilibrium point that maximizes throughput. Future work could include more realistic simulation experiments with microscopic simulations, comparison with other control methods from literature, and establishing stability properties.

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