Moving Horizon Estimation for Large-scale Urban Networks

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Abstract

Control of large-scale urban traffic presents a significant challenge. Usually, traffic states are assumed to be measured without any noise in the literature on control of large-scale urban networks (LSUNs). This is an unrealistic assumption from the application point of view, since in a real setting measurements are affected by noise to varying degrees. As it is impossible to have noise-free measurements, traffic management schemes for LSUNs require development of state estimation methods for filtering out this noise to obtain reliable information on the traffic states. Moving horizon estimation (MHE) stands out as a state estimation method suitable for traffic applications with nonlinear system dynamics, constraints on the state, and access to demand profiles. A moving horizon of past measurements is considered in MHE, where the state estimation problem is cast as an optimization problem which is solved repeatedly in real-time. We develop in this study an MHE scheme for LSUNs, with a prediction model based on the macroscopic fundamental diagram (MFD). To provide an application setting, a perimeter control based MPC scheme is also developed, which works together with the proposed MHE scheme for large-scale traffic management. Simulations of a two-region LSUN with congested traffic conditions are included, where state estimation performance of the developed MHE scheme is evaluated together with the control performance of the MPC. Results suggest that using MHE leads to better performance.

Keywords

Moving horizon estimation, traffic state estimation, large-scale urban networks, macroscopic fundamental diagram.

1 Introduction

Modeling, estimation, and control of large-scale urban traffic networks present considerable challenges. Inadequate infrastructure and coordination, low sensor coverage, spatiotemporal propagation of congestion, and the uncertainty in traveler choices contribute to the difficulties faced when creating realistic models and designing effective traffic estimation and control schemes for urban networks. Although considerable research has been directed towards designing efficient real-time traffic management schemes in the last decades, estimation and control of heterogeneously congested large-scale urban networks remains a challenging problem.

Traffic modeling and control studies for urban networks usually focus on microscopic models keeping track of link-level traffic dynamics with control strategies using local information. Based on the linear-quadratic regulator (LQR) problem, traffic-responsive urban control (TUC) (Diakaki *et al.,* 2002) and its extensions (Aboudolas *et al.,* 2010, Kouvelas *et al.,* 2011) represent a multivariable feedback regulator approach for network-wide urban traffic control. Although TUC can deal with oversaturated conditions via minimizing and balancing the relative occupancies of network links, it may not be optimal for heterogeneous networks with multiple pockets of congestion. Inspired by the max pressure routing scheme for wireless networks, many local traffic control schemes have been proposed for networks of signalized intersections (see Varaiya (2013), Kouvelas *et al.* (2014), Wongpiromsarn *et al.* (2012), Zaidi *et al.* (2015)), which involve evaluations at each intersection requiring information exclusively from adjacent links. Although the high accuracy of microscopic traffic models is desirable for simulation purposes, the increased model complexity results in complications for control, whereas local control strategies might not be able to operate properly under heavily congested conditions, as they do not protect the congested regions upstream. Another disadvantage of sophisticated local controllers is that they might require detailed information on traffic states, which are difficult to estimate or measure.

Literature on traffic state estimation mainly focuses on freeway networks: A mixture Kalman filter based on the cell transmission model (Daganzo, 1995) is proposed in Sun *et al.* (2003). In Wang and Papageorgiou (2005), an extended Kalman filter is proposed for real-time state and parameter estimation for a freeway network, the dynamics of which is described by the METANET model (Messner and Papageorgiou, 1990). Mihaylova *et al.* (2007) develops a particle filtering framework for a second order freeway traffic model that is efficiently parallelizable. Yuan *et al.* (2012) reports the superiority of Lagrangian state estimation formulations over the Eulerian case using extended Kalman filters for the Lighthill-Whitham and Richards (LWR) model. There is also some literature on urban traffic state estimation: Pueboobpaphan and

Nakatsuji (2006) design an unscented Kalman filter based on a kinematic wave model modified for urban traffic. A combined approach via integrating the Kalman filter with advanced data fusion techniques is taken by Kong *et al.* (2009) for urban network state estimation. Nantes *et al.* (2016) propose a data fusion based extended Kalman filter for urban corridors based on the LWR model. Interestingly, even though there is considerable literature on model-based state estimation for freeways, there are very few works on comparable techniques for urban road networks.

An alternative to local real-time traffic control methods is the two layer hierarchical control approach. At the upper layer, the network-level controller optimizes network performance via regulating macroscopic traffic flows through interregional actuation systems (e.g., perimeter control), whereas at the lower layer the local controllers regulate microscopic traffic movements through intraregional actuation systems (e.g., signalized intersections). The macroscopic fundamental diagram (MFD) of urban traffic is a modeling tool for developing low complexity aggregated dynamic models of urban networks, which are required for the design of efficient network-level control schemes for the upper layer. It is possible to model an urban region with roughly homogeneous accumulation (i.e., small spatial link density heterogeneity) with an MFD, which provides a unimodal, low-scatter, and demand-insensitive relationship between accumulation and trip completion flow (Geroliminis and Daganzo, 2008).

The concept of MFD with an optimum accumulation was first proposed by Godfrey (1969), and its existence was recently verified with dynamic features and real data by Geroliminis and Daganzo (2008). Control strategies based on MFD modeling and using perimeter control type actuation (i.e., manipulating transfer flows between neighboring regions) have been proposed by many researchers for single-region (Daganzo, 2007, Keyvan-Ekbatani *et al.*, 2012, Gayah *et al.*, 2014, Haddad and Shraiber, 2014) and multi-region (Haddad and Geroliminis, 2012, Aboudolas and Geroliminis, 2013) urban areas. Application of the MPC technique to the control of urban networks with MFD modeling also attracted recent interest. Geroliminis *et al.* (2013) design a nonlinear MPC for a simple two-region urban network equipped with a perimeter control system. Haddad *et al.* (2013) develop an MPC scheme for the cooperative control of a mixed transportation network consisting of a freeway and two urban regions. Hajiahmadi *et al.* (2015) generalize the two-region MFD network model of Geroliminis *et al.* (2013) to that of an R-region network, and propose hybrid MPC schemes for an urban network equipped with both perimeter control systems and switching signal timing plans. Ramezani *et al.* (2015) develop a model capturing the dynamics of heterogeneity and design a hierarchical control system with MPC on the upper level. More detailed literature reviews in MFD-based modeling and control can be found in Saberi and Mahmassani (2012) and Yildirimoglu *et al.* (2015).

Although there is considerable literature on traffic estimation (especially for freeway networks), combined estimation and control for heterogeneously congested large-scale urban networks remains an open problem. In this paper we propose integrated schemes for real-time optimization based estimation and control for urban networks with MFD-based modeling. Specifically, a moving horizon estimation (MHE) scheme is designed for a two-region urban network, which is coupled with a model predictive control (MPC) with perimeter control actuation. Estimation and control performances of the proposed scheme are evaluated via simulations with a congested scenario under varying levels of uncertainty in demands and measurement noise. Results indicate substantial improvement in control performance with the use of the proposed MHE scheme, suggesting considerable value for practice.

2 Modeling of Urban Networks

Consider a heterogeneous urban road network $\mathcal{R}$ that can be partitioned into 2 homogeneous regions, i.e., $\mathcal{R} = \{1, \ldots, n_R\}$ with $n_R = 2$. Each region has a well-defined outflow MFD, defined via $G_I(N_I(t))$ (veh/s), which is the outflow at accumulation $N_I(t)$. The demand for trips in region I with destination J is $Q_{IJ}(t)$ (veh/s), whereas $N_{IJ}(t)$ (veh) is the accumulation in region I with destination J, and $N_I(t)$ (veh) is the total accumulation in region I, at time t; $I, J \in \mathcal{R}$; $N_I(t) = \sum_{J \in \mathcal{R}} N_{IJ}(t)$. Between the two regions 1 and 2 there exists perimeter controls $U_{12}(t)$ and $U_{21}(t) \in [0, 1]$, that can manipulate the transfer flows. The dynamics of the 2-region MFDs network is (Geroliminis *et al.,* 2013):

$$\dot{N}_{11}(t) = Q_{11}(t) + U_{21}(t) \cdot M_{21}(t) - M_{11}(t) \tag{1a}$$

$$\dot{N}_{12}(t) = Q_{12}(t) - U_{12}(t) \cdot M_{12}(t) \tag{1b}$$

$$\dot{N}_{21}(t) = Q_{21}(t) - U_{21}(t) \cdot M_{21}(t) \tag{1c}$$

$$\dot{N}_{22}(t) = Q_{22}(t) + U_{12}(t) \cdot M_{12}(t) - M_{22}(t), \tag{1d}$$

where the $M_{II}(t)$ and $M_{IJ}(t)$ terms express the exit and transfer flows, which can be expressed as follows:

$$M_{II}(t) = \frac{N_{II}(t)}{N_I(t)} G_I(N_I(t)) \quad \forall I \in \mathcal{R} \tag{2a}$$

$$M_{IJ}(t) = \frac{N_{IJ}(t)}{N_I(t)} G_I(N_I(t)) \quad \forall I \in \mathcal{R},\ J \in \mathcal{R} \setminus \{I\}. \tag{2b}$$

All trips inside a region are assumed to have similar trip lengths (i.e., the origin and destination of the trip does not affect the distance traveled by a vehicle). Simulation and empirical results (Geroliminis and Daganzo, 2008) suggest the possibility of approximating the MFD by an asymmetric unimodal curve skewed to the right (i.e., the critical accumulation N_I^{cr}, for which $G_I(N_I(t))$ is at maximum, is less than half of the jam accumulation N_I^{jam} that puts the region in gridlock). Thus, $G_I(N_I(t))$ can be expressed using a third-order polynomial in $N_I(t)$:

$$G_I(N_I(t)) = A_I N_I^3(t) + B_I N_I^2(t) + C_I N_I(t), \tag{3}$$

where A_I, B_I, and C_I are estimated parameters.

The inflow demand terms $Q_{IJ}(t)$ are assumed to have uncertainty, which we model as follows:

$$Q_{IJ}(t) = max(D_{IJ}(t) + W_{IJ}(t), 0) \quad \forall I, J \in \mathcal{R}, \tag{4}$$

where $D_{IJ}(t)$ expresses a known average demand profile and $W_{IJ}(t)$ is the associated demand noise with zero mean and normal distribution, i.e., $W_{IJ}(t) \sim \mathcal{N}(0, \sigma_W^2)$, with σ_W^2 specifying the demand noise variance. Furthermore, it is assumed that the network is equipped with sensors that can measure the accumulations $N_{IJ}(t)$, for which the measurement noise can be similarly modeled as follows:

$$Y_{IJ}(t) = max(N_{IJ}(t) + V_{IJ}(t), 0) \quad \forall I, J \in \mathcal{R}, \tag{5}$$

where $Y_{IJ}(t)$ is the measurement on $N_{IJ}(t)$ whereas $V_{IJ}(t)$ is the associated measurement noise with zero mean and normal distribution, i.e., $V_{IJ}(t) \sim \mathcal{N}(0, \sigma_V^2)$, with σ_V^2 specifying the measurement noise variance.

3 Estimation and Control of Large-scale Networks

3.1 Moving Horizon Estimation

We formulate the problem of finding the N_{IJ} and W_{IJ} values that minimize the discrepancy between measurements and the prediction model, for a moving time horizon extending a fixed length into the past, as the following discrete time nonlinear MHE problem:

$$\begin{aligned}
\underset{N,\,W}{\text{minimize}}\ & \sum_{k=-N_e}^{-1} W(k)_Q^2 + \sum_{k=-N_e}^{0} Y(k) - N(k)_R^2 \\
\text{subject to}\quad & \text{for } k = -N_e, \ldots, 0 : \\
& \qquad Y(k) = \tilde{Y}(t-k) \\
& \text{for } k = -N_e, \ldots, -1 : \\
& \qquad N(k+1) = f_e(N(k), U(k), D(k), W(k)),
\end{aligned} \tag{6}$$

where N_e is the estimation horizon, k is the time step, Q and R are weighting matrices on demand and measurement noise, respectively, $\tilde{Y}(t)$ is the measurement taken at sampling instant t, $Y(k)$, $N(k)$, $U(k)$, $D(k)$, and $W(k)$ are the vectors containing all $Y_{IJ}(k)$, $N_{IJ}(k)$, $U_{IJ}(k)$, $D_{IJ}(k)$, and $W_{IJ}(k)$ terms, respectively, whereas f_e is the time discretized version of Eq. (1).

3.2 Model Predictive Control

We formulate the problem of finding the U_{IJ} values that minimize TTS as the following discrete time economic nonlinear MPC problem:

$$\begin{aligned}
\underset{N,\,U}{\text{minimize}}\quad & T \cdot \sum_{k=0}^{N_p-1} N(k)_1 \\
\text{subject to}\quad & N(0) = \tilde{N}(t) \\
& \text{for } k = 0, \ldots, N_p - 1 : \\
& \qquad N(k+1) = f_p(N(k), U(k), D(k)) \\
& \qquad 0 \le N_{IJ}(k) \quad \forall I, J \in \mathcal{R} \\
& \qquad N_I(k) \le N_I^{\text{jam}} \\
& \qquad U_{\min} \le U(k) \le U_{\min},
\end{aligned} \tag{7}$$

where k and T are the time step and sample time, respectively, N_p is the prediction horizon, t is the current sampling instant in time and $\tilde{N}(t)$ is the state estimate computed by the MHE at that instant, $N(k)$, $U(k)$, and $D(k)$ are vectors containing all $N_{IJ}(k)$, $U_{IJ}(k)$, and $D_{IJ}(k)$ terms, respectively, f_p is the time discretized version of Eq. (1) (with the assumption that all W_{IJ} terms are 0, as these have zero mean but are unknown to the MPC), whereas $U_{\min}$ and $U_{\min}$ are the bounds on the perimeter control inputs.

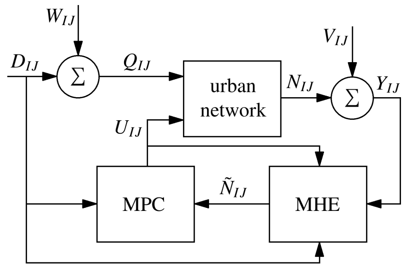

Figure 1: Architecture of combined MHE-MPC scheme.

3.3 Integrated Moving Horizon Estimation and Model Predictive Control

For the combined accumulation state estimation and perimeter control of large-scale urban networks, we propose a scheme integrating MHE and MPC (see Fig. 1). In this scheme, the MHE has access to information on noisy measurements Y_{IJ} of accumulation states N_{IJ}, perimeter control inputs U_{IJ}, and average inflow demands D_{IJ}, for a fixed time horizon (i.e., N_e) into the past. Using these, at time t, the MHE computes the accumulation state estimate $\tilde{N}_{IJ}(t)$ by solving the problem (6). The state estimate is then used by the MPC, together with information on average inflow demand profiles D_{IJ} for a fixed time horizon (i.e., N_p) into the future, to compute the perimeter control inputs $U_{IJ}(t)$ via solving the problem (7). The control inputs are applied to the urban network, completing the feedback loop.

4 Case Studies

4.1 Simulation Setup

All simulations are conducted on a 2-region urban network (see **??**), with the simulation model given in Eq. (1) for representing the reality. The regions have the same MFD, with the parameters $A_I = 4.133 \cdot 10^{-11}$, $B_I = -8.282 \cdot 10^{-7}$, $C_I = 0.0042$, jam accumulation $N_I^{\text{jam}} = 10^4$ (veh), critical accumulation $N_I^{\text{cr}} = 3.4 \cdot 10^3$ (veh), maximum outflow $G(N_I^{\text{cr}}) = 6.3$ (veh/s), which are consistent with the MFD observed in a part of downtown Yokohama (see Geroliminis and Daganzo (2008)). Standard deviations of the demand and measurement noise are chosen as $\sigma_W = 2$ veh/s and $\sigma_V = 1000$ veh, representing presence of substantial uncertainty, whereas the weighting matrices of the MHE are chosen as $Q = (1/\sigma_W^2)I$ and $R = (1/\sigma_V^2)I$, to reflect the fact that the stage cost terms related to the demand and measurement noises should be weighted inversely proportional to the associated amount of uncertainty (that is, e.g., the measurements should be trusted more if the measurement noise has a lower variance). Sample time is $T = 60$ s and the simulation length is $T_{\text{exp}} = 200$ (in number of discrete time steps), whereas estimation and prediction horizons are chosen as $N_e = N_p = 20$, specifying a horizon of 20 minutes and a simulation length of 200 minutes.

The metrics for evaluating the MHE and MPC schemes are *root mean square of estimation error* (RMS_{EE}) (veh) and TTS (veh·s), respectively, which are defined for a single simulation experiment as follows:

$$\text{RMS}_{\text{EE}} = \sqrt{\frac{\sum_{t=1}^{T_{\text{exp}}} \sum_{I \in \mathcal{R}} \sum_{J \in \mathcal{R}} (N_{IJ}(t) - \tilde{N}_{IJ}(t))^2}{n_{\mathcal{R}}^2 \cdot T_{\text{exp}}}} \tag{8}$$

$$\text{TTS} = T \cdot \sum_{t=1}^{T_{\text{exp}}} \sum_{I \in \mathcal{R}} \sum_{J \in \mathcal{R}} N_{IJ}(t). \tag{9}$$

4.2 Estimation and Control Performance under Congested Conditions

The network is empty at the beginning and experiences increasing inflow demands with time (see Fig. 2). We compare three different cases, each with the MPC described with Eq. (7): (a) MPC-1 has access to information on the actual traffic state N_{IJ}, representing the case with perfect measurements. (b) MPC-2 has access to state estimates $\tilde{N}_{IJ}$ computed by the MHE given with Eq. (6), representing the case with combined MHE and MPC. (c) MPC-3 has access to the measurements Y_{IJ}, representing the case of MPC without MHE. The results are given in Fig. 3, which indicate that, expectedly, MPC-1 has the best performance owing to the perfect measurements, while MPC-3 performs badly due to the noisy measurements and uncertain demands. It is perhaps more interesting to note that MPC-2 performs fairly close to MPC-1, suggesting that MPC is able to perform well even under situations with severe demand uncertainty and measurement noise, if it is coupled with MHE.

4.3 Sensitivity of Estimation and Control Performance to Changes in Uncertainty Levels

To study the effect of changing the levels of uncertainty present in the inflow demand profiles and the intensity of measurement noise, a series of simulation experiments are conducted with

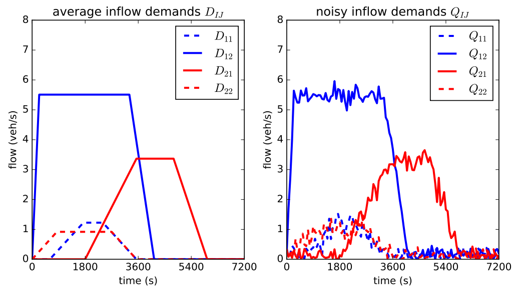

Figure 2: Average and noisy inflow demand profiles of the congested scenario.

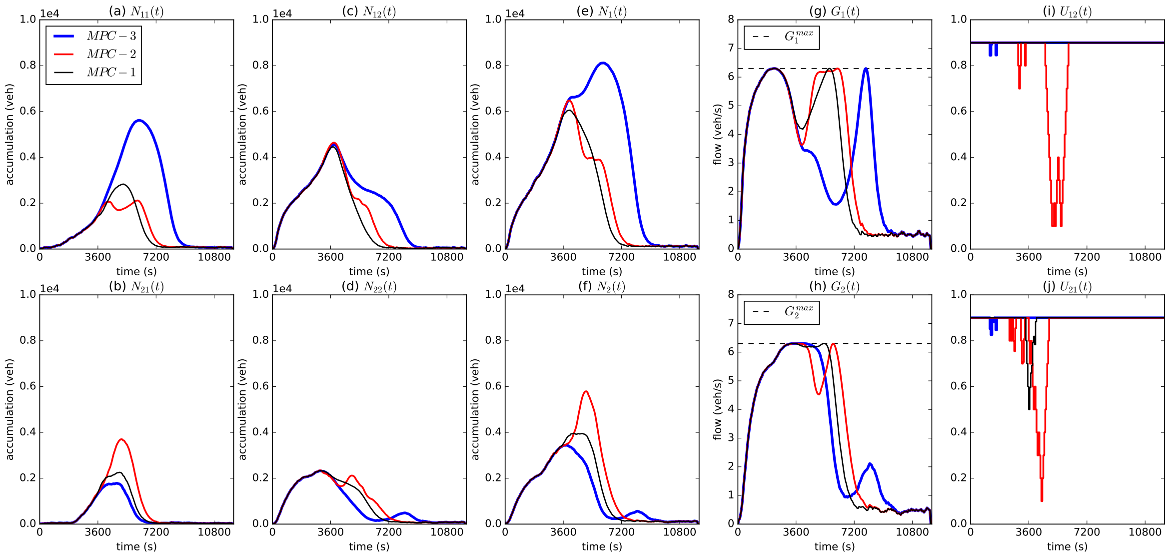

Figure 3: Results of the congested scenario for MPC-1 (perfect measurement), MPC-2 (MPC with MHE), MPC-3 (MPC without MHE): (a)–(d) Accumulations, (e)–(f) regional accumulations, (g)–(h) regional outflows, (i)–(j) perimeter control inputs.

varying the associated standard deviation values, namely σ_W and σ_V, for the combined MHE-MPC scheme. The results, given in Table 1 and Table 2, present the estimation (RMS_{EE}) control (TTS) performance for the various levels of uncertainty. These results indicate that although the MHE is somewhat sensitive to changes in noise intensity, MPC is still able to perform well for a wide range of uncertainty levels, suggesting substantial potential for practical applications, where uncertainty in demand information and measurement noise are unavoidable.

5 Conclusion

In this paper we proposed a combined MHE-MPC scheme for the integrated estimation and control of heterogeneously congested large-scale urban networks, the dynamics of which can be described by MFD of urban traffic. We compared the proposed scheme with MPC schemes

Table 1: Estimation performance ($\mathrm{RMS_{EE}}$) of the combined MHE-MPC scheme for various values of σ_W and σ_V.

σ_W (veh/s)	σ_V = 250 veh	σ_V = 500 veh	σ_V = 750 veh	σ_V = 1000 veh
0.5	516.9	563.2	646.6	672.9
1	526.9	614.5	666.3	732.6
1.5	586.2	642.4	763.2	812.6
2	653.2	727.8	783.9	842.0

Table 2: Control performance (TTS, $\times 10^7$) of the combined MHE-MPC scheme for various values of σ_W and σ_V.

σ_W (veh/s)	σ_V = 250 veh	σ_V = 500 veh	σ_V = 750 veh	σ_V = 1000 veh
0.5	3.95	3.99	4.10	4.09
1	4.07	4.14	4.14	4.23
1.5	4.15	4.20	4.50	4.46
2	4.24	4.16	4.54	4.55

having perfect and noisy measurements, and found that the proposed MHE-MPC scheme is able to perform very close to the case with perfect measurements. Further analysis related to the performance sensitivity under varying levels of uncertainty indicated that although estimation performance of MHE itself is somewhat sensitive to varying intensity of uncertainty, the combined MHE-MPC scheme is able to perform well for a wide range of uncertainty levels. Thus, the proposed MHE-MPC scheme shows substantial potential for practical applications in traffic management systems for large-scale urban road networks.

Future work could include: (a) Evaluating the proposed MHE-MPC scheme via more detailed simulation models to represent the traffic reality. (b) Comparison of the proposed MHE scheme with existing traditional approaches (e.g., extended Kalman filter).

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