Model Predictive Control of Urban Networks with Perimeter Control and Route Guidance

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Abstract

Recent studies on empirical data demonstrated the existence of macroscopic fundamental diagram (MFD), which expresses an aggregated model of traffic dynamics linking accumulation and output flow of an urban region. Employing MFD in modeling of urban networks opens up possibilities to introduce a new generation of real-time traffic control structures and improve mobility. Although there are studies on using MFD modeling for designing control structures with perimeter control actuation, the potential of actuation via route guidance still needs to be explored. This paper proposes a traffic control scheme based on nonlinear model predictive control (MPC), an advanced control technique based on real-time repeated optimization, for improving mobility in urban networks, integrating perimeter control and route guidance type actuation. Perimeter control operates at region boundaries and manipulates the transfer flows between regions, whereas route guidance system recommends drivers at a region with a specific destination which neighboring region to go next. Two simpler controllers are designed for comparison: 1) perimeter control MPC and 2) route guidance MPC. Performance of the controllers are evaluated via simulations on a 7-region network for a high demand scenario. Results suggest substantial potential in improvement of urban mobility through use of route guidance based MPC schemes.

Keywords

Macroscopic fundamental diagram, perimeter control, route guidance, model predictive control.

1 Introduction

Modeling and control of large-scale urban traffic networks present considerable challenges. Inadequate infrastructure and coordination, spatiotemporal propagation of congestion, and the uncertainty in traveler choices contribute to the difficulties faced when creating realistic models and designing effective traffic control schemes for urban networks. Although considerable research has been directed towards designing efficient real-time traffic control schemes in the last decades (see Papageorgiou *et al.* (2003) for a review), control design for heterogeneously congested large-scale urban networks remains a challenging problem.

Traffic modeling and control studies for urban networks usually focus on microscopic models keeping track of link-level traffic dynamics with control strategies using local information. Based on the linear-quadratic regulator (LQR) problem, traffic-responsive urban control (TUC) Diakaki *et al.* (2002) and its extensions Aboudolas *et al.* (2010), Kouvelas *et al.* (2011) represent a multivariable feedback regulator approach for network-wide urban traffic control. Although TUC can deal with oversaturated conditions via minimizing and balancing the relative occupancies of network links, it may not be optimal for heterogeneous networks with multiple pockets of congestion. Inspired by the max pressure routing scheme for wireless networks Tassiulas and Ephremides (1992), many local traffic control schemes have been proposed for networks of signalized intersections (see Varaiya (2013), Kouvelas *et al.* (2014), Wongpiromsarn *et al.* (2012), Zaidi *et al.* (2015)), which involve evaluations at each intersection requiring information exclusively from adjacent links. Although the high accuracy of microscopic traffic models is desirable for simulation purposes, the increased model complexity results in complications for control, whereas local control strategies might not be able to operate properly under heavily congested conditions, as they do not protect the congested regions upstream. Another disadvantage of sophisticated local controllers is that they might require detailed information on traffic states, which are difficult to estimate or measure.

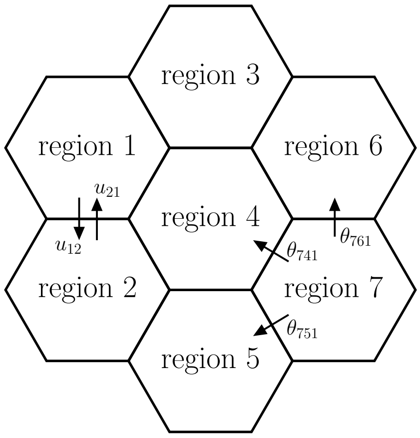
An alternative to local real-time traffic signal control methods is the two layer hierarchical control approach. At the upper layer, the network-level controller optimizes network performance via regulating macroscopic traffic flows through interregional actuation systems (e.g., perimeter control), whereas at the lower layer the local controllers regulate microscopic traffic movements through intraregional actuation systems (e.g., signalized intersections). The macroscopic fundamental diagram (MFD) of urban traffic is a modeling tool for developing low complexity aggregated dynamic models of urban networks, which are required for the design of efficient network-level control schemes for the upper layer. It is possible to model an urban region with roughly homogeneous accumulation (i.e., small spatial link density heterogeneity) with an

MFD, which provides a unimodal, low-scatter, and demand-insensitive relationship between accumulation and trip completion flow Geroliminis and Daganzo (2008).

The concept of MFD with an optimum accumulation was first proposed by Godfrey (1969), and its existence was recently verified with dynamic features and real data by Geroliminis and Daganzo (2008). Control strategies based on MFD modeling and using perimeter control type actuation (i.e., manipulating transfer flows between neighboring regions) have been proposed by many researchers for single-region Daganzo (2007), Keyvan-Ekbatani *et al.* (2012), Gayah *et al.* (2014), Haddad and Shraiber (2014) and multi-region Haddad and Geroliminis (2012), Aboudolas and Geroliminis (2013) urban areas. Application of the MPC technique to the control of urban networks with MFD modeling also attracted recent interest. Geroliminis *et al.* (2013) design a nonlinear MPC for a simple two-region urban network equipped with a perimeter control system. Hajiahmadi *et al.* (2015) generalize the two-region MFD network model of Geroliminis *et al.* (2013) to that of an R-region network, and propose hybrid MPC schemes for an urban network equipped with both perimeter control systems and switching signal timing plans. Haddad *et al.* (2013) develop an MPC scheme for the cooperative control of a mixed transportation network consisting of a freeway and two urban regions. Ramezani *et al.* (2015) develop a model capturing the dynamics of heterogeneity and design a hierarchical control system with MPC on the upper level. More detailed literature reviews in local traffic control, MFD modeling, and MFD based control can be found in Aboudolas *et al.* (2010), Kouvelas *et al.* (2014), and Ramezani *et al.* (2015). These recent studies on perimeter control based MPC schemes for urban networks do not explore any opportunity for manipulating the routes of the drivers through feedback control via actuation with route guidance systems. Although there are also some recent attempts at developing traffic control schemes with route guidance capability Yildirimoglu *et al.* (2015), Hajiahmadi *et al.* (2013), the integration of perimeter control and route guidance type actuation still remains unexplored.

In this work we first introduce a new urban network model capable of expressing aggregated traffic dynamics via MFDs, while at the same time avoiding cyclic behavior (i.e., prohibiting vehicles from flowing back and forth between neighboring regions), which is where it differs from similar MFD-based urban network models. Furthermore, we design network-level nonlinear MPC schemes for a heterogeneous urban traffic network with a given partition into homogeneous regions (see Fig. 1), each with a well-defined MFD. We extend upon earlier works that propose perimeter control actuation based MPC schemes for urban networks by integrating the route guidance type actuation in the MPC formulation. Simulation results on a 7 region network show that using route guidance has substantial potential in improving urban mobility, and using both perimeter control and route guidance leads to further improvement.

Figure 1: Schematic of an urban network with 7 regions.



2 Modeling of Urban Networks

Consider an urban network partitioned into several homogeneous regions, each with a well-defined MFD. The network $\mathcal{R}$ is a set of R regions, i.e., $\mathcal{R} = \{1, 2, \ldots, R\}$. Let $q_{ij}(t)$ (veh/s) denote the exogenous traffic flow demand generated in region i with destination region j, $n_{ij}(t)$ (veh) the vehicle accumulation in region i with destination region j, and $n_i(t)$ (veh) the total accumulation in region i, at time t; $i, j \in \mathcal{R}$; $n_i(t) = \sum_{j\in\mathcal{R}} n_{ij}(t)$. We assume that between each pair of neighboring regions i and j there exists perimeter controllers $u_{ij}(t)$ and $u_{ji}(t)$ (–), $u_{ij}(t), u_{ji}(t) \in [0, 1]$, that can manipulate the transfer flows between regions i and j. The traffic flow conservation equation of an R-region MFDs network is (largely based on the work of Ramezani *et al.* (2015) and Yildirimoglu *et al.* (2015))

$$n_{ij}(t+1) = n_{ij}(t) + T_s\left(q_{ij}(t) - \sum_{h\in\mathcal{N}_i} u_{ih}(t)\hat{M}_{ihj}(t) + \sum_{h\in\mathcal{N}_i;h\neq j} u_{hi}(t)\hat{M}_{hij}(t)\right) \quad \forall i, j \in \mathcal{R}, \tag{1}$$

where t (–) and T_s (s) are the simulation time step counter and the simulation sampling period, respectively, with $t \in \mathbb{N}$, $\mathcal{N}_i$ is the set of regions neighboring region i, and $\hat{M}_{ihj}(t)$ (veh/s) is the effective (i.e., constrained by the boundary capacity between regions) transfer flow from region i with destination j through the next immediate region h, which is calculated as (Yildirimoglu *et al.,* 2015):

$$\hat{M}_{ihj}(t) = \min\left(M_{ihj}(t), c_{ih}(n_h(t))\frac{n_{ij}(t)\theta_{ihj}(t)}{\sum_{k\in\mathcal{R}} n_{ik}(t)\theta_{ihk}(t)}\right) \tag{2}$$

where $M_{ihj}(t)$ and $\theta_{ihj}(t)$ denote the transfer flow and the percentage of outflow, respectively, from region i to destination region j through the next immediate region h, and $c_{ih}(n_h(t))$ is the boundary capacity between regions i and h that depends on the accumulation in region h. The boundary capacity constraint can be omitted in the prediction model inside MPC for computational advantage. The physical reasoning of this omission is that (i) the boundary capacity decreases for accumulations much larger than the critical accumulation, and (ii) the controller will not allow the regions to have accumulations close to gridlock (Haddad *et al.,* 2013). With this omission, the transfer flow $M_{ihj}(t)$ is used in the prediction model given in Eq. (1) instead of the effective transfer flow $\hat{M}_{ihj}(t)$, and is calculated corresponding to the ratio between accumulations as

$$M_{ihj}(t) = \theta_{ihj}(t)\frac{n_{ij}(t)}{n_i(t)}G_i(n_i(t)), \tag{3}$$

where $G_i(n_i(t))$ (veh/s) is the trip completion flow for region i at accumulation $n_i(t)$, defining the MFD of region i. It is assumed that all trips inside a region have similar lengths (i.e., the distance traveled per vehicle inside a region does not depend on the origin and destination of the trip). Simulation and empirical results (Geroliminis and Daganzo, 2008) suggest that the MFD can be approximated by an asymmetric unimodal curve skewed to the right (i.e., the critical accumulation, which maximizes $G_i(n_i(t))$, is smaller than half of the jam accumulation $n_{i,\text{jam}}$, which puts the region in gridlock). Thus, $G_i(n_i(t))$ can be expressed with a third-order polynomial in the variable $n_i(t)$, e.g., $G_i(n_i(t)) = a_i n_i^3(t) + b_i n_i^2(t) + c_i n_i(t)$, where a_i, b_i, and c_i are estimated parameters.

The urban network model given by Eq. (1) does not prohibit vehicles from flowing back and forth between neighboring regions (i.e., it permits cyclic behavior), leading to unrealistic results when this model is used in simulations for representing the reality. This is especially evident in cases with route guidance only actuation, where the controller tries to emulate perimeter control via cyclic routes. The modeling contribution of the paper for avoiding this cyclic behavior is given in the following dynamic equation

$$n_{ogij}(t+1) = n_{ogij}(t) + T_s\left(\sum_{f\in\mathcal{N}_g\cup g\setminus\{i,j\}} u_{gi}(t)\theta_{ofgij}(t)M_{ofgj}(t) - \sum_{h\in\mathcal{N}_i\setminus\{o,g\}} u_{ih}(t)\theta_{ogihj}(t)M_{ogij}(t)\right), \tag{4}$$

where $n_{ogij}(t)$, M_{ogij}, and $\theta_{ogihj}(t)$ denote the accumulation in region i, transfer flow from region i, and the percentage of outflow from region i, respectively, with origin region o, immediately preceding region g, destination region j, and, for $\theta_{ogihj}(t)$, through the next immediate region h. $\mathcal{N}_g$ is the set of regions neighboring region g. The model in Eq. (4) prohibits cyclic behavior and thus is a more realistic representation of urban network dynamics. Being a complex model that can capture more realistic dynamics, this model is used as the simulation model in the simulation case studies (for representing the reality), whereas the model given by Eq. (1) is used as the prediction model in MPC for computational advantage.

3 Model Predictive Control Design

We formulate the problem of finding the u_{ij} and θ_{ihj} values minimizing the total network delay as the following finite horizon constrained optimal control problem:

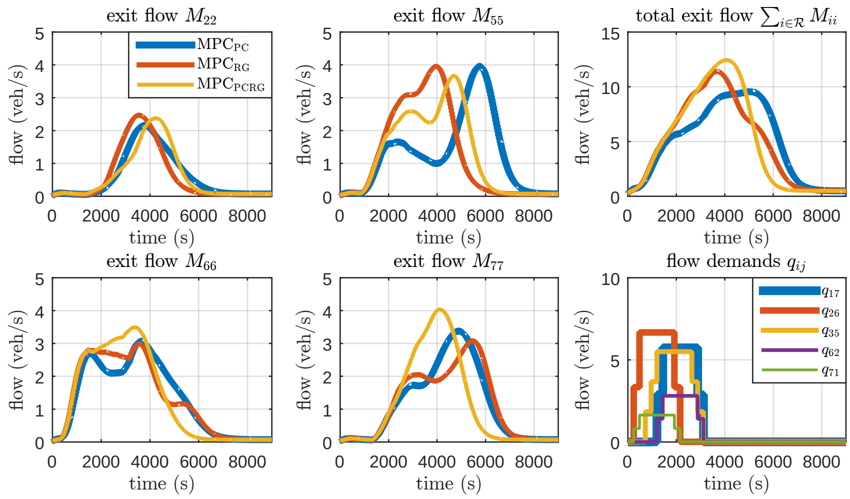
$$\begin{aligned}
\underset{u_{ih},\ \theta_{ihj}}{\text{minimize}} \quad & T_c \textstyle\sum_{k=0}^{N_p} \sum_{i=1}^{R} n_{i,k} \\
\text{subject to} \quad & \forall i, j \in \mathcal{R}:\ n_{ij,0} = n_{ij}(t_c), \\
& \forall k \in \{0, \ldots, N_p - 1\}: \\
& \quad \text{Model equations (1) and (3)},\ n_{i,k} = \textstyle\sum_{j \in \mathcal{R}} n_{ij,k},\ 0 \le n_{i,k} \le n_{i,\text{jam}} \\
& \quad u_{\min} \le u_{ih,k} \le u_{\max},\ 0 \le \theta_{ihj,k} \le 1,\ i \neq j,\ h \in \mathcal{N}_i,\ \textstyle\sum_{h \in \mathcal{N}_i} \theta_{ihj,k} = 1,\ i \neq j \\
& \quad \text{if } N_c \le k \le N_p - 2:\ u_{ih,k+1} = u_{ih,k},\ h \in \mathcal{N}_i,\ \theta_{ihj,k+1} = \theta_{ihj,k},\ i \neq j, h \in \mathcal{N}_i
\end{aligned} \tag{5}$$

where t_c (–) and T_c (s) are the control time step counter and the control sampling time, N_p and N_c are the prediction and control horizons in control time steps, and $u_{\min}$ and $u_{\max}$ are the bounds on the perimeter control inputs. Due to the nonlinear prediction model, Eq. (5) is a nonconvex nonlinear optimization problem, which can be solved using nonlinear optimization methods. We propose three MPC schemes: (a) MPC_{PC} optimizes over u_{ih}, while drivers choose their own routes (in simulations, this is captured by calculating θ_{ihj} via a route choice algorithm employing a logit model), (b) MPC_{RG} optimizes over θ_{ihj}, while u_{ih} inputs are fixed to $u_{\max}$, (c) MPC_{PCRG} optimizes over both u_{ih} and θ_{ihj}.

4 Simulation Results

Simulation results for a high demand scenario, on an urban network with 7 regions with the structure given in Fig. 1, is given in Figure 2 for comparing the three MPC schemes. Each region is assumed to have the same MFD, with the MFD parameters $a_i = 1.4877 \cdot 10^{-7}/3600$, $b_i = -2.9815 \cdot 10^{-3}/3600$, $c_i = 15.0912/3600$, jam accumulation $n_{i,\text{jam}} = 10^4$ (veh), critical accumulation $n_{i,\text{cr}} = 3.4 \cdot 10^3$ (veh), and maximum outflow $G_i(n_{i,\text{cr}}) = 6.3$ (veh/s), which are consistent with the MFD observed in Yokohama (see Geroliminis and Daganzo (2008)). Prediction and control horizons are chosen as $N_p = 6$ and $N_c = 2$, length of the simulation experiment is 280 (in number of simulation steps), whereas simulation and control sampling times are $T_s = 30$ s and $T_c = 240$ s, giving an effective prediction horizon of 24 minutes and an effective simulation length of 140 minutes. Bounds of the perimeter control commands are $u_{\min} = 0.1$ and $u_{\max} = 0.9$.

Figure 2: Exit flows, total exit flow and flow demands for a high demand scenario comparing the three controllers.



The network exit flow $\sum_{i\in\mathcal{R}} M_{ii}$ graph in Fig. 2, shows the difference in time it takes for the controllers to clear the network, suggesting that MPC_{PCRG} has the best performance in terms of total network delay, while MPC_{RG} and MPC_{PC} are thus second and last in performance, respectively, suggesting potential for substantial improvement in mobility via route guidance based schemes, and also showing that using perimeter control together with route guidance can result in further improvement.

5 Conclusions and Future Work

We proposed in this paper (a) a novel urban network model with cyclic behavior prohibition feature that allows more realistic MFD-based urban traffic simulations for route guidance based control schemes, (b) various network-level nonlinear MPC schemes, using perimeter control and route guidance type actuators, with prediction models based on MFD-modeling of urban networks. Via simulations, we demonstrated the possibility of substantial improvement in urban mobility through the use of route guidance based MPC schemes, and further improvement via using perimeter control and route guidance actuation together.

Future work will include (a) more detailed simulation experiments for evaluating the performance of the controllers in the face of uncertainty in flow demands and noise in measurements of accumulations, (b) a sensitivity analysis for assessing how the controllers perform in the face of varying levels of driver compliance to route guidance commands θ_{ihj}.

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