An analytical approach for modelling tolled bus lanes

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Conference paper STRC 2014

STRC | 14th Swiss Transport Research Conference
Monte Verità / Ascona, May 14-16,2014

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April 2014

Abstract

High-occupancy vehicle (HOV) and bus lanes have been widely implemented in several countries as a measure to reduce highway congestion, or provide bus priority, in urban or suburban transportation networks. While general use lanes can be congested, if flows in bus or HOV lanes are low, these lanes can be severely underutilized. If ordinary vehicles, such as cars, are allowed in the bus lane at certain rates and in exchange of some toll, however, an optimized operation for all lanes could be achieved. Hence, the aim of this paper is to provide a generalized solution regarding the allowance rate of cars in the bus lane, as well as the toll rates that should be applied.

Keywords

Bus lanes – traffic operations – tolled lanes

1. Introduction

The intensive urbanization rates of past decades lead to a high demand for transport infrastructure. To this end, extended highway networks have been developed across urban centers in order to serve the growing demand. However, the high demand-growth rate soon created congested conditions in highways. Therefore, special measures such as bus and high-occupancy vehicle (HOV) lanes had to be taken so as to relieve congestion and reduce overall travel times.

As there is a high probability of underutilization of these lanes when bus or HOV flows are low, there have been efforts to optimize the entire operation. Extended research and studies on that topic led to the implementation of the first high-occupancy/toll (HOT) lane in California in 1995 (Kwon and Varaiya, 2008). Many other examples have followed since then (Barker and Polzin, 2007; Turnbull, 2008). This concept dedicates a lane primarily for HOVs, although it can also be used by single occupant vehicles willing to pay a toll (Dahlgren, 2002; Daganzo and Cassidy, 2008). Nowadays, managed lanes and dynamic toll systems are research topics which, thanks to the advanced technological possibilities, can be tested in practice (Yin and Lou, 2009). Furthermore, solutions for more efficient use of bus lanes by cars have also been suggested in previous research (Guler and Cassidy, 2012).

Despite such advancements, allowing car drivers to use bus lanes on highways by paying a toll still constitutes a challenging and innovative concept. This idea would not only optimize the utilization of the bus lanes' capacity – while producing a monetary gain for the managing authorities – but could also reduce the delays experienced by drivers in the general purpose lanes. This idea differs from HOT and managed lanes in two ways: firstly, the arrival rate of buses is fixed, and secondly, a strict restriction for not disturbing bus operations is present.

In this paper, we focus on a highway segment between two toll stations, which serve also as check points for the arrival and discharge rates of the vehicles on each lane. This segment consists of several general purpose lanes and one physically separated bus lane, so as to prohibit lane changes within the segment. The case of a simple continuous segment with no bus stops, where bus overtaking is not permitted, is investigated. In the course of the study, two subcases are considered, in which the time gain and, consecutively, the implemented toll are either time-dependent or constant. Analytical expressions concerning the allowance rate of cars in the bus lane are derived by combining elements of the fundamental, the queuing and the time-space diagrams of each variant. Then, a formula regarding the upper bound of the toll is defined, so that the number of drivers that decide to switch lanes becomes optimal.

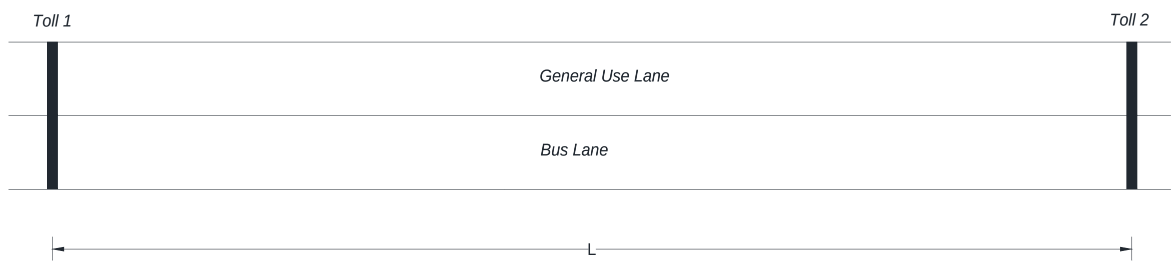
2. Theoretical analysis

To describe the operation of tolled bus lanes, the flow of cars allowed to use the bus lane and the toll charged need to be determined. To do so, a theoretical analysis of the system is carried out in this section using tools from traffic flow theory.

A multiple-lane highway consists of one bus lane (*bl*) and ρ general use lanes (*gl*), which are physically separated (Figure 1). Let us consider a segment between two consecutive toll stations of length L kms. A certain number of cars can be allowed in the bus lane, as long as the bus operation is not affected. Cars can enter or exit the bus lane only at the toll stations. It is important to mention that in this scenario there are no bus stops within the segment under investigation.

Suppose that there is a bottleneck downstream of toll station 2 leading to queued conditions on the general use lanes. In this scenario, it is assumed that the back of the queue lies between toll station 1 and 2. Once the average speed of cars on the general use lanes drops below a certain threshold it becomes attractive for some drivers to divert to the bus lane in exchange of paying a toll. The cars that divert will have to adapt to the conditions of the bus lane, i.e. the slower speed of the bus.

Figure 1 Layout of analysis section



Buses have a fixed headway of h_b hours and the time-space diagram for buses and cars travelling on the bus lane is shown in Figure 2. In this figure, the blue lines represent the buses and the black lines represent the cars.

It is assumed that traffic operations on a single general or bus use lane can be described with a fundamental diagram as shown in Figure 3. The capacity on a single general use lane is Q_{max} veh/hour/lane, the free flow speed is v_f km/hour and the jam density is k_j veh/km/lane. On the general use lanes, there exists a bottleneck with capacity Q_{gl} veh/hour/lane. On the bus lane, due to the slow speed of the bus, the capacity is reduced to Q_{bl} veh/hour, with a corresponding speed of v_b km/hour equal to the bus speed.

Figure 2 Time-Space diagram (x-t) ; blue lines: buses, black lines: cars

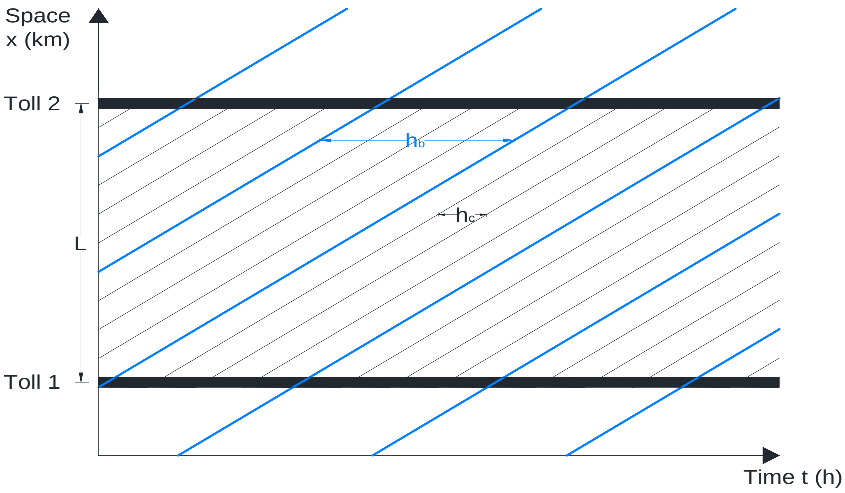
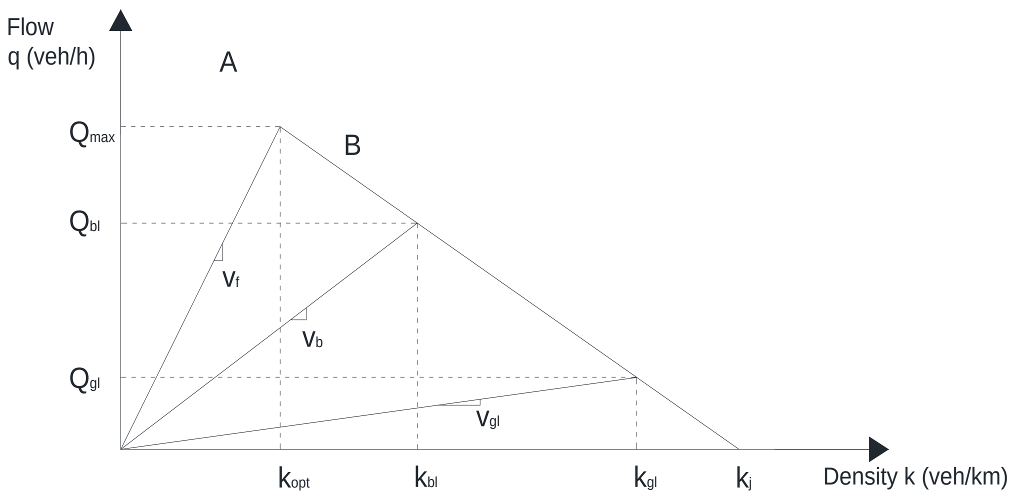


Figure 3 The fundamental diagram for a single general use or bus use lane



From the properties of the triangular fundamental diagram, the reduced capacity on the bus lane, Q_{bl} , can then be found as follows:

$$Q_{bl} = \frac{Q_{max} \cdot v_f \cdot v_b \cdot k_j}{Q_{max} \cdot (v_f - v_b) + v_f \cdot v_b \cdot k_j} \qquad (1)$$

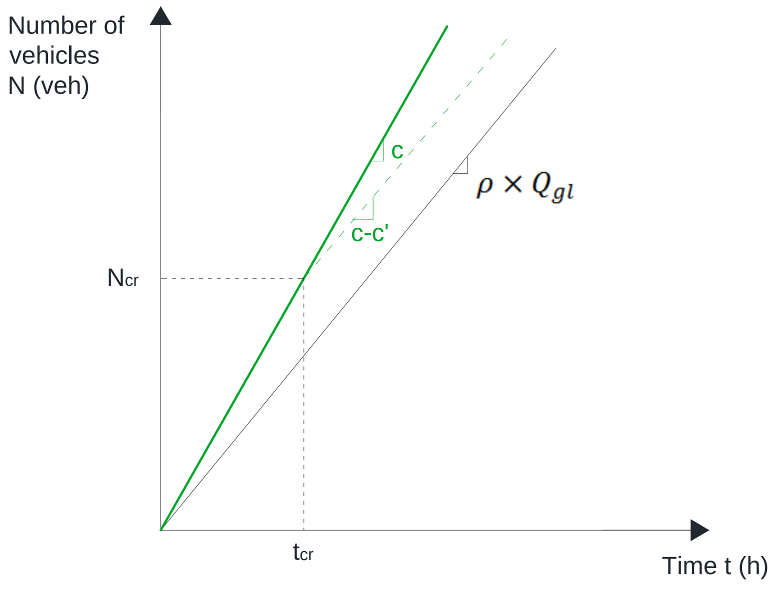
Then, the queuing diagram for the general use lanes is shown in Figure 5. In this figure, the cumulative number of virtual arrivals (i.e., time cars would have arrived to toll station 2 if there were no queuing) are shown as a thick solid line with slope c depicting the demand rate of cars ($c < \rho \times Q_{max}$), and the departures from toll station 2 are shown as a thin solid line with slope $\rho \times Q_{gl}$. It is assumed that at the critical moment, t_{cr}, the N_0^{th} car decides to divert from the general use lane to the bus lane since the travel time on the general use lane becomes longer than on the bus lane. The dashed line in Figure 5 depicts the virtual arrival curve to toll station 2 on the general use lanes after some drivers start diverting to the bus lane. The slope of this line is $c - c'$, where c' (veh/hour) depicts the flow of cars on the bus lane. The following expressions concerning the N_{gl}^{th} car of the general use lane and the N_{bl}^{th} of the bus lane can be derived from the queuing diagram:

$N_{Tot}(t) = c \cdot t$ expressing demand in general use lane (2)

$N_{gl}(t) = (c - c') \cdot t + c' \cdot t_{cr}$ expressing cars remaining in general use lane

$N_{bl}(t) = c' \cdot (t - t_{cr})$ expressing cars remaining entering the bus lane

Figure 4 Queuing diagram for general use lanes



To determine t_{cr}, the travel time on the general use lanes should be compared to the travel time on the bus lane. Before diverting the travel time in the general use lane for the N^th^ car is equal to the sum of the free flow travel time ($fftt$) and the delay (d_{gl}) which can be expressed as in Equation 3.

$$t_{gl}(N) = fftt + d_{gl} = \frac{L}{v_f} + \frac{N}{\rho \times Q_{gl}} - \frac{N}{c} \tag{3}$$

To be reduce the traffic demand from the general use lanes, a flow of c' veh/hour can be allowed in the bus lane such that $c' = \min\left(c - \rho \times Q_{gl}, Q_{bl} - \frac{1}{h_b}\right)$. This means that the demand in the general use lane is at least $\rho \times Q_{gl}$ once drivers start to divert, and the demand for the bus lane does not exceed its capacity. This allows for the queue in the general use lanes to either remain constant, or to continue to grow but at a lower rate. If more cars are allowed to divert the queues on the general use lanes would start to diminish, eliminating the incentive for drivers to use the bus lane. Here, two distinct subcases can be identified:

i) The demand curve after t_{cr} has a slope equal to $\rho \times Q_{gl}$ ($c - c' = \rho \times Q_{gl}$)
ii) The demand curve after t_{cr} has a slope greater than $\rho \times Q_{gl}$ ($c - c' > \rho \times Q_{gl}$)

For these two subcases, a time gain (TG) can be determined based on the difference between the travel time on the general use lanes and the travel time on the bus lane.

First subcase

When the demand curve after t_{cr} has a slope equal to $\rho \times Q_{gl}$ the travel time on the general use lane remains constant with time. In this case, when the travel time on the general use lane (t_{gl}) becomes equal to the travel time on the bus lane ($t_{bl} = L/v_b$) drivers could be expected to divert to the bus lane. If drivers divert at this time, no toll can be charged since the time gain would be 0. However, this time can be thought of as the minimum value for the critical time, t_{cr}^{min}.

$$t_{cr}^{min} = \frac{L \cdot Q \cdot (v_f - v_b)}{v_f \cdot v_b \cdot (c - NGL \times Q_{gl})} \tag{4}$$

The actual time at which cars are allowed into the bus lane, t_{cr}, can then be determined based on a toll determined in advance. In this case the toll is pre-determined and could be based on different factors such as expected total income or level of service on the general use lanes. Using this toll, the time at which drivers will start diverting onto the bus lane can be calculated as in Equation 5.

$$t_{cr} = t_{cr}^{min} + toll/\varphi \quad (5)$$

where φ is the value of time in money units/unit time.

Second subcase

When the demand curve after t_{cr} has a slope greater than $\rho \times Q_{gl}$, the demand on the general use lanes continues to exceed the capacity. Hence the travel time on the general use lanes continues to increase with time. In this case, the travel time on the general use lanes can be calculated as the sum as the free flow travel time, and the delay which can be obtained from the queuing diagram.

$$t_{gl} = fftt_{gl} + d_{gl} = \frac{L}{v_f} + N \times \left(\frac{1}{\rho \times Q_{gl}} - \frac{1}{c - c'} \right) + t_{cr} \times \frac{c'}{c - c'} \quad (6)$$

where the time gain depends on the car number, N.

So the time gain and the toll are now time-dependent in the sense that they differ for every next car. Nevertheless they can be calculated as follows:

$$TG = t_{gl} - t_{bl} = fftt_{gl} + d_{gl} - fftt_{bl} = \frac{L \cdot (v_b - v_f)}{v_f \cdot v_b} + N \times \left(\frac{1}{\rho \times Q_{gl}} - \frac{1}{c - c'} \right) + t_{cr} \times \frac{c'}{c - c'} \quad (7)$$

$$toll = TG \cdot \varphi = \left\{ \frac{L \cdot (v_b - v_f)}{v_f \cdot v_b} + N \times \left(\frac{1}{\rho \times Q_{gl}} - \frac{1}{c - c'} \right) + t_{cr} \times \frac{c'}{c - c'} \right\} \cdot \varphi \quad (8)$$

3. Conclusions

In this paper the optimization problem of the operation of a highway, which includes bus lanes, has been treated and analytical expressions for potential toll implementation have been derived. The toll rates have been determined such that car drivers have an incentive to use the bus lane to bypass the queue in the general purpose lanes, but also to limit the number of users of the tolled bus lane in order not to disturb bus operations. One simple case, which has been divided in two subcases that vary according to the desired result, has been investigated.

It has been proven that an improvement of the operation of general use lanes can be achieved, decreasing the delay both for drivers that choose to divert to the bus lane and for the ones remaining in the general use lane, while the bus operation is not disturbed. In addition to that, both constant and time-dependent tolls, depending on the strategy that the managing authority would choose, could be regarded as beneficial solutions.

However, further research on the topic needs to be conducted. Other variants, in which a bus stop is included within the segment, for example, should be investigated in order to provide a better overview of the problem and the suggested alternative solutions and to highlight the benefit potentials for drivers in terms of time gain and for managing authorities in financial terms.

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