

# **Explaining the households' decision on car ownership and use using an approach based on an indirect utility function**

## **Abstract**

In this paper, I present a model that can be viewed as an extension of the traditional Tobit model. As opposed to that specific model, ours also accounts for the fixed costs of car ownership. That extension is needed since being carless is an option for many households in societies that have a good system of public transportation, the main reason being that carless households wish to save the fixed costs of car ownership. So far, no existing model can adequately map the impact of these fixed costs on car ownership. By use of the modelling framework I propose I overcome this limitation. My model is based on an indirect utility function corresponding to a linear Marshallian demand function and includes the fixed costs of car ownership. By use of this model I can evaluate the effect of policies intended to influence household behaviour with respect to car ownership, which can be of great interest to policy makers. My model makes it possible to compute the effect of policies such as taxes on fuel or on car ownership on both the share of carless households and the average driving distance. I calibrated the model using data on Swiss private households in order to be able to forecast responses to policies. By use of these data I will also calibrate a model which is based on a Marshallian demand function of a log-linear form as well as a model based on a direct utility function and I will compare the resulting elasticities.

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## **1. Introduction**

My model is based on the Marshallian and indirect utility as used in the paper by Dubin and McFadden “An econometric analysis of residential electric appliance holdings and consumption” (1984), where they present the so-called “Discrete-Continuous choice model” for the first time. This model captures a joint decision of deciding on one type of capital good and the intensity of using this capital good. Examples of such decisions are the choice of type of heating system and then the choice of room temperature that will define the energy costs as examined by Dubin and McFadden (1984) or the choice of car type and the annual mileage driven. Unfortunately, their model cannot capture the case where households decide not to use a car. Our model shall fill this gap. Note, that our model will not be able to capture the households decisions on different car types and only covers the decision between being carless and owning a car and the intensity of using it.

In the following, I first describe what I assume on household behaviour when they decide on the choice and use of a type of capital good. I then present the microeconomic demand system that maps this behaviour. Next, I will state the assumptions on the error term and I will show how the parameters of this model can be estimated. I will then show how the elasticities of the driving demand and the car ownership can be computed. At last, I present the empirical results using data on Swiss private households.

## **2. Introduction of the model**

In this paragraph, I present the microeconomic demand system, which should map the households' behaviour. The basic idea behind this model is that the household computes its utility for two cases. Case (a), when it decides not to own a car and spends all its income on other goods and case (b) when it decides to own a car and to drive a certain annual distance. The household will then choose the case that yields the higher utility.

I start by describing how the Marshallian demand function relates to the level of utility given the case (b) where the household chooses to own a car and bear the fixed costs of car ownership. Given the choice of owning a car it is assumed that a household chooses the annual driving distance  $x_2$  that provides the highest utility, given its income  $y$  net the fixed costs of car ownership  $k_2$  and the marginal driving costs  $p_2$ . Good one  $x_1$  is a composite good containing all goods but the capital good. The price of this composite good  $p_1$  is regarded to be numéraire and thus the utility  $x_1$  provides can also be regarded as the utility of the remaining income after having paid for the expenditures for the car ownership and its use irrespective if this remaining income has been spent entirely on the composite good or if it has been saved. Note, that I assume that only car driving provides utility but not car

ownership itself. I assume that the households decision corresponds to a microeconomic modelling framework that corresponds to a linear Marshallian demand function (3):<sup>1</sup>

$$V_b = \left( \frac{\alpha}{\beta} + \beta \cdot (y - k_2) + \alpha p_2 + \gamma s + \varepsilon \right) \cdot e^{-\beta p_2}, \quad (1)$$

$$X_2 = x_2(p_2, y - k_2, \alpha, \beta, \gamma s, \varepsilon_c) = \alpha p_2 + \beta(y - k_2) + \gamma s + \varepsilon. \quad (2)$$

Note, that the indirect utility function (1) and the Marshallian demand function (2) are linked by Roy's identity and there is only a very limited set of Marshallian demand functions for which the corresponding utility function is of a known and explicit form that allows for a quick computation.<sup>2</sup> Here,  $s$  reflects socio-demographic variables of the household. The random variable  $\varepsilon$  contains unobserved socio-demographic variables. Relevant unobserved household attributes could be the preference for car driving or a disability that prevents one member of the household from using public transportation. The Marshallian demand function (2) describes what driving distance the household would choose in the case (b) and what utility level (1) it would reach in the case.

In the alternative case (a) the household chooses not to own a car. In this case the complete income  $y$  is available to the household and the demand for car driving is zero by definition. The utility level of this case cannot be computed straight forward, since the direct utility function is unknown. Thus, I have first to compute the marginal cost of driving demand  $p_2$  that corresponds to a Marshallian demand of driving (2) of zero for the case where the household's budget is equal to its total income  $y$  and then plug this value into the indirect utility function (1):<sup>3</sup>

$$V_a = \frac{\alpha}{\beta} \cdot e^{\beta(\beta y + \gamma s + \varepsilon)/\alpha}. \quad (3)$$

The household will now decide for being carless if  $V_a > V_b$  and the random variable  $X_2$  is defined as follows:

<sup>1</sup> In fact, the formulas (1) and (2) only hold, if  $\varepsilon > \varepsilon_0$ , where  $x_2(p_2, y, \alpha, \beta, \gamma, \varepsilon_0) = 0$ . Later on I will show, that it is not necessary to consider the case  $\varepsilon < \varepsilon_0$  and thus I do not show how (1) and (2) would be in that case.

<sup>2</sup> Roy's identity is defined as follows:  $x_2(p_1, p_2, y) = -\partial v(p_1, p_2, y) / \partial p_2 / \partial v_i(p_1, p_2, y) / \partial y$ . For a proof that (1) corresponds to (2), see Appendix A1. Note that also Dubin and McFadden (1984:349) used this linear form for their Discrete-continuous Choice Model, see Dubin and McFadden (1984:349). The outline of this proof can be found in Hausman (1981:668). In Hausman (1981) other functional forms of the Marshallian demand function can also be found.

<sup>3</sup> In the first step I set (2) to zero:  $0 = \alpha p_2 + \beta y + \gamma s + \varepsilon$ . Note that the fixed costs are now zero  $k_2 = 0$ , since the household does not own a car any more. Solving for  $p_2$  yields:  $p_2 = -(\beta y + \gamma s + \varepsilon) / \alpha$ . Plugging this solution in (1) yields:

$$V_a = \left( \frac{\alpha}{\beta} + \beta \cdot y - \alpha((\beta y + \gamma s + \varepsilon) / \alpha) + \gamma s + \varepsilon \right) \cdot e^{-\beta} = \frac{\alpha}{\beta} \cdot e^{-\beta(\beta y + \gamma s + \varepsilon) / \alpha}.$$

$$X_2 = \begin{cases} V_a < V_b : 0 \\ V_a \geq V_b : \alpha p_2 + \beta(y - k_2) + \gamma s + \varepsilon \end{cases} \quad (4)$$

where  $V_a$  and  $V_b$  are defined in (1) and (2) and  $\varepsilon$  is normal distributed with zero mean and standard deviation  $\sigma$ . From this follows the following probability function of  $X_2$ :

$$f(z) = f(z, y, p_2, e_c, \theta) = \begin{cases} z = 0 : \Phi\left(\frac{e_c}{\sigma}\right) \\ z > x_{2,c} : \phi\left(\frac{z - \alpha p_2 - \beta y - \delta s}{\sigma}\right) \end{cases} \quad (5)$$

with  $\theta = (\alpha, \beta, \delta, \sigma)$ ,

$$\text{where } P_a(y, k_2, p_2, \alpha, \beta, \gamma) = P(V_a > V_b) = P(X_2 < x_{2,c}) = P(\varepsilon < e_c) = \Phi\left(\frac{e_c}{\sigma}\right), \quad (5a)^4$$

$$\text{and } g(x_{2,c}) = 0, \quad x_{2,c} > 0, \quad \text{with } g(x_2) = \frac{\alpha}{\beta} \cdot (e^{\beta/\alpha(x_2 + \beta k_2)} - 1) - x_2,$$

$$\text{and } e_c = x_{2,c} - \alpha p_2 - \beta y - \delta s.^5$$

$F(\bullet)$  is the cumulated density function corresponding to the random variable  $\varepsilon$ , e.g.  $F(e) = \Phi(e/\sigma)$  for the case, where  $\varepsilon$  is normal distributed with zero mean and standard deviation  $\sigma$ , where  $\Phi(\bullet)$  is the cdf of the standard normal distribution.

Since  $g(x_2)$  does not contain neither  $y$  and nor  $p_2$  the critical driving distance  $x_{2,c}$  does not depend on  $y$  or  $p_2$ . The critical relative unobserved preference  $\varepsilon_c$  can be directly computed from  $x_{2,c}$ ,  $\varepsilon_c = x_{2,c} - \alpha p_2 - \beta(y - k_2) - \gamma s$ . It is easy to show, that  $\varepsilon_c$  depends negatively on  $y$  and positively on  $p_2$ . This means, that the probability of not owning decreases with  $y$  but increases with  $p_2$ .<sup>6</sup> This is intuitive, since households with a higher income are more likely to own a car and bear the fixed costs of car ownership and car ownership becomes less likely if driving costs increase. Note, that the critical driving distance  $x_{2,c}$  reflects the distance below which no household would drive if it had a car and thus following this model there is no observation  $x_2$  in the interval  $0 < x_2 < x_{2,c}$  possible. This is also economically intuitive: No households would buy and hold a car when it plans only to drive 2000 kilometres per year for instance, since for this case it would be much cheaper to use taxi services or to rent a car for some specific trips. It is also intuitive that the driving distance depends positively on  $k_2$ , since if the fixed cost are high, it would even be relatively even cheaper to use taxi services or to rent a car for some specific trips when intending to drive only a few miles per year.

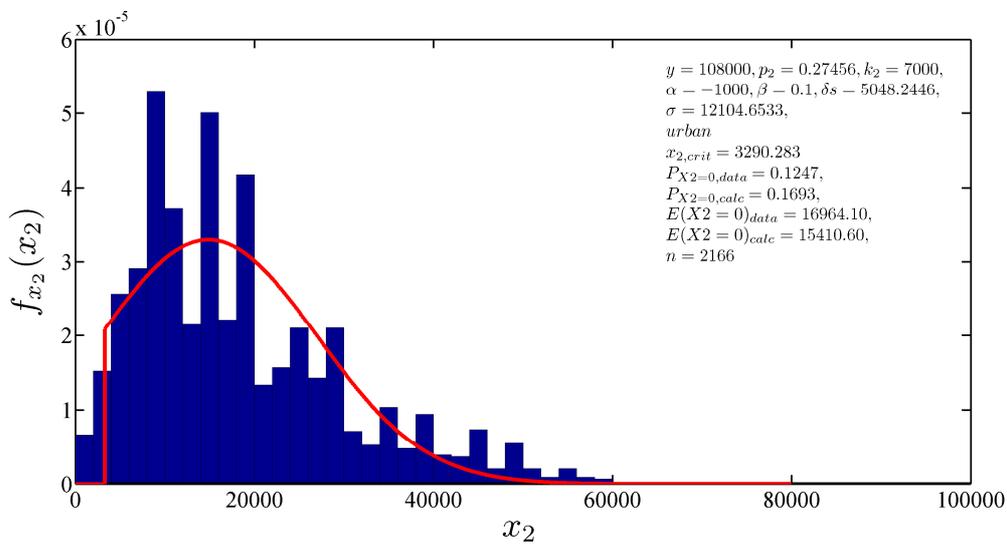
<sup>4</sup> For a proof, see appendix A4.

<sup>5</sup> Note that it is straight forward to express  $(V_a - V_b)(e_c)$  as a function of  $x_{2,c}$  instead,  $(V_a - V_b)(x_{2,c})$ , by using (2).

<sup>6</sup> Recall, that the parameter  $\alpha$  is negative, since the amount of kilometres driven  $x_2$  depends negatively on the driving costs  $p_2$ .

### 3. Estimation of the model parameters

The parameters shall be estimated such that the model explain the real word data as good as possible. To do so, the Maximum Likelihood estimation (MLE) method will be applied. Unfortunately, the fact that following this model there is no observation  $x_2$  in the interval  $0 < x_2 < x_{2,c}$  possible causes a problem to apply the MLE method. The reason is that if any observation  $x_2$  is in the in the interval  $0 < x_2 < x_{2,c}$  the MLE function will be zero too and thus in such cases the function cannot be maximized. Note, that in real data there are always some observations in the interval  $0 < x_2 < x_{2,c}$  because of households who misreport or simply have an unusual preference for owning a car but using it intensively. The following table shows that such households exist but are very rare.



**Figure 1:** Empirical and theoretical distribution for urban household with an income of 108,000CHF<sup>7</sup>

Formula (5a) shows that the minimum driving distance only depends only on the parameters  $\alpha$  and  $\beta$  and the fixed cost  $k_2$  of car ownership. Thus the estimation of the parameters  $\alpha$  and  $\beta$  play a crucial role with respect to the minimum driving distance. Changing the parameters  $\alpha$  and  $\beta$  has an impact on whether some observations  $x_2$  fall in the interval  $0 < x_2 < x_{2,c}$  which results to that the MLE function will be zero and thus the parameters cannot be estimated. I circumvent this problem by the applying following estimation routine:

1. Choose values for  $\alpha$  and  $\beta$ .
2. Compute  $x_{2,c}$  for each observation  $n$   $x_{2,c}$  .
3. Eliminate all observations where  $0 < x_2 < x_{2,c}$  .

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<sup>7</sup> Note, that the height of the bars are normalized by factor  $1/n$  so that the total surface of all bars equals one.

4. Estimate the parameters  $\delta$  and  $\sigma$  by MLE conditional on  $\alpha$  and  $\beta$ . Compute a penalty function that depends a) positively on the proportion of eliminated datasets, b) positively on the relative error of the difference between the average simulated proportion of carless households, c) positively on the actual proportion of carless households and d) on the difference between the average simulated expectation value of driving demand and the actual average driving distance. Note that the actual proportion of carless households and the actual average driving distance refer to the measures based on the dataset after eliminating the observations according to Step 3.
5. Repeat Steps 1 - 5 for a number of different values for  $\alpha$  and  $\beta$  (grid search). Choose values  $\alpha$  and  $\beta$  so that the lowest value of the penalty function is yielded.

For MLE estimation I use the following log ML function:

$$L(x, y, p_2, s, \theta) = \sum_{i=1}^n \log(f(x_i)) = \sum_{i=1}^n I(x_i = 0) \cdot \log\left(\Phi\left(\frac{e_{c,i}}{\sigma}\right)\right) + I(x_i > 0) \cdot \log\left(\phi\left(\frac{x_i - \alpha p_{2i} - \beta y_i - \delta s_i}{\sigma}\right)\right), \quad (6)$$

where  $e_{c,i}$  and  $\theta$  are defined in (5).

As the penalty function I chose

$$Q = \left(\frac{P_{sim} - P_{real}}{P_{real}}\right)^2 + c_1 \cdot \left(\frac{E_{sim}(X_2) - \text{mean}(x_2)}{\text{mean}(x_2)}\right)^2 + c_2 \cdot \left(\frac{\# \text{ elim. observations}}{\text{size of initial datasets}}\right)^2, \quad (7)$$

where  $P_{sim}$  is the average of the simulated probabilities,  $P_{real}$  is the actual proportion of carless households in the dataset,  $E_{sim}(X_2)$  is the average of the simulated expectation values of driving distance and  $\text{mean}(x_2)$  is the mean of the actual driving distance in the dataset. Expressions “ $(E_{sim}(X_2) - \text{mean}(x_2))/\text{mean}(x_2)$ ” and “ $(P_{sim} - P_{real})/P_{real}$ ” are the relative errors of the average of the simulated values, which could be called “replication errors”. Here “dataset” relates to the dataset after eliminating the dataset where  $0 < x_2 < x_{2,c}$ . Expression “# elim. observations/size of initial datasets” corresponds to the percentage of eliminated datasets with respect to the initial number of datasets. Parameters  $c_1$  and  $c_2$  are weighting parameters. I chose  $c_1 = 1$ , which means that both types of replication errors should be weighted about equally, and  $c_2 = 0.5$ . The latter choice yields a proportion of 3.5% of the observations that was eliminated; see the section on “Results”. Considering the fact that some households may simply have stated a too low driving distance, a proportion of 3.5% of the eliminated observations seems to be quite reasonable.

Note, that due to the assumption of  $X_2$  being normally distributed (2) the computation of the expectation value of  $X_2$  and thus the computation of  $E_{sim}(X_2)$  is straight forward.<sup>8</sup>

$$E_{sim}(X_2) = \frac{1}{n} \sum_{i=1}^n E(X_{2,i}), \quad (8)$$

$$\text{with } E(X_{2,i}) = (\alpha p_{2,i} + \beta y_i + \delta s_i) \cdot \left( 1 - \Phi \left( \frac{x_{2,c} - \alpha p_2 - \beta y - \delta s}{\sigma} \right) \right) - \sigma \cdot \phi \left( \frac{x_{2,c,i} - \alpha p_{2,i} - \beta y_i - \delta s_i}{\sigma} \right).$$

That the expectation value  $E(X_{2,i})$  is the great advantage of choosing the error term  $\varepsilon$  to be normally distributed. Thus, a lot of computation time can be saved, since  $E(X_{2,i})$  has to be computed for each observation and grid point  $(\alpha, \beta)$ , when applying the estimation routine.

#### 4. Data

The data I used to estimate the parameters is the micro-census data on the travel behaviour of Swiss households, Swiss Federal Statistical Office SFSO (2006). 33,000 households were interviewed. The dates of the interviews were more than less evenly distributed over the year 2005. This dataset contains a vast number of information on traveling behaviour, ownership of cars, motorbikes and bicycles, and information on the households. Since the purpose of this study is to investigate fuel demand, I will use the information on total kilometres driven by cars. Since in the present model I do not consider the choice of different car types, I will use the total annual kilometres driven by all households as a proxy for fuel demand. Since I am basically interested in the effect of fuel prices on the distance travelled and the decision of whether or not to own one or several cars, I will only use the household variables that appeared to have the most important impact on travel distance or fuel demand in other models.<sup>9</sup> In this case, I will only use the income and the place of living residence as explanatory variables, namely whether the households live in a rural area or in a non-rural area, which I denote “urban areas”. As in Bhat (2008)<sup>10</sup>, I choose the price of the composite good  $x_1$  to be numéraire.<sup>11</sup> Since  $p_1$  is one, amount  $x_1$  is nothing but the income  $y$  minus the amount spent of driving,  $k_2 + p_2 x_2$ , since I assume that households spend all of their income and do not save anything. Of course, it is a simplification to assume that households will spend all of their income on consumption, but no data on savings is available in the dataset. Furthermore, also savings can be regarded as to provide utility since to have savings allows for future consumption and also contributes also to a positive feeling when having some money aside.

<sup>8</sup> For a proof see appendix A9.

<sup>9</sup> Note that when including more explanatory variables, the resulting elasticities do not change much, see Appendix xy.

<sup>10</sup> “If an outside good is present, label it as the first good which now has a unit price of one,” Bhat (2008: 290). Note that Bhat denotes an “outside good” as a good that is always chosen.

<sup>11</sup> This is reasonable since the price of a composite good is a price index, and a price index is scale-free.

	$\text{mean}(x=0)$	$\text{mean}(x)$	$\text{sdev}(x)$	$\text{min}(x)$	$\text{max}(x)$
$x_2$	0.1890	13,890	12,195	0	59,731
$y$	--	80,187	43,373	18,000	228,000
$k_2$	--	7,000	0	7,000	7,000
$p_2$	--	0.2745	0.0036	0.2692	0.2838
<i>rural</i>	0.7717	0.2283	0.2745	0	1

**Table 1:** Summary statistics of the data, SFSO (2006)<sup>12</sup>

Note, that observation of the driving distance  $x_2$  with more than 60,000 kilometres were eliminated, since these observations are considered for being wrongly reported. Note, that those observations would strongly influence the MLE value and thus would cause biases in the parameter estimation.

## 5. Results

I used two specifications. The first was only with the sociodemographic variable “rural”. The reason for that is that I wanted to be able to split the data into the segments “rural”/“urban” for each income category. This will allow to compare the probability function to the histogram of observations of the driving distance  $x_2$  for each such segment and thus to have some intuitive idea on the model’s fit to the data. In order to see, if the resulting elasticities change if more explanatory variables  $s$  were added, I added the most relevant sociodemographic variable “number of people in the household” to the model. For the parameters  $a$  and  $b$  I chose grids with ranges  $a = -80,000, \dots, -1,000$  and  $b = 0.01, \dots, 0.2$ .<sup>13</sup>

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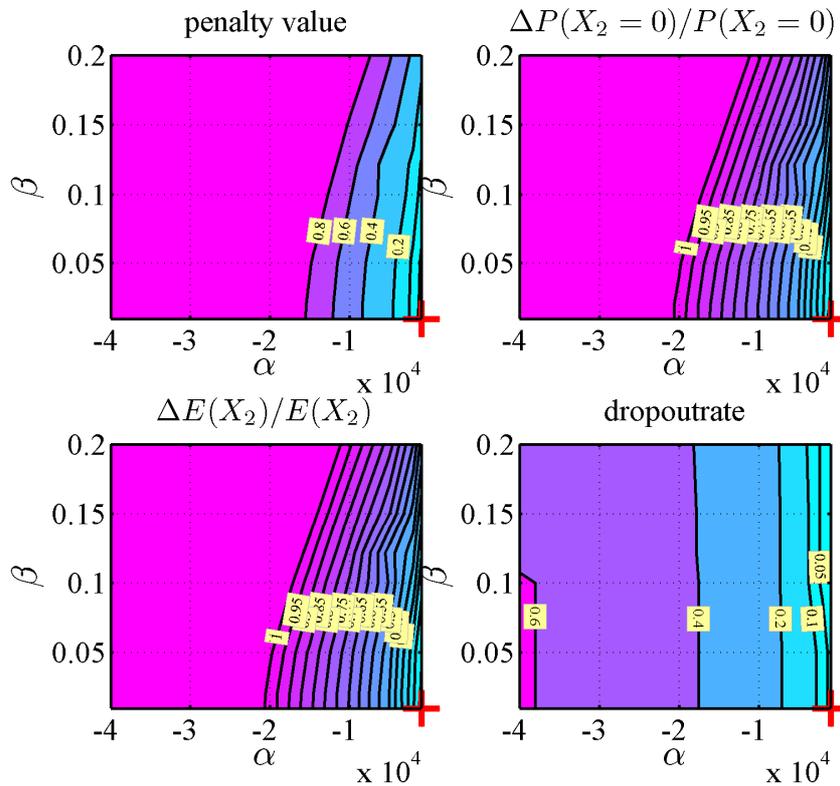
<sup>12</sup> Note, that all economic variables are in CHF and  $x_2$  is measured in kilometres. The fixed costs  $k_2$  and the variable costs  $p_2$  of a car correspond to a cost estimation of a standard, midsize car, TCS (2007). The variable costs  $p_2$  are computed as follows:  $p_2 = 0.1601 + 0.0778 \cdot p_{fuel}$ , where  $p_{fuel}$  is the average fuel price of the twelve month prior to the date a household was reporting its data. Note further, that the values in the column “ $\text{mean}(x=0)$ ” reflects  $\text{mean}(x_2=0)$  which is equal to the share of carless households and the share of households living in a rural area,  $\text{mean}(rural)$ .

<sup>13</sup> The values of the grid were:  $a = (-1000, -2000, -5000, -10000, -15000, -20000, -26000, -30000, -50000, -80000)$  and  $b = (0.010, 0.020, 0.050, 0.100, 0.121, 0.150, 0.200)$ .

	“small model”	“incl. #pers”
$\alpha$	-1000	-1000.0
$\beta$	0.1	0.05000
$\tilde{\delta}_0$	5048.2	2639.1
$\tilde{\delta}_{\text{rural}}$	5676.4	4519.0
$\tilde{\delta}_{\text{\#pers}}$	-	2735.0
$\sigma$	12104.7	12133.7
<b>n</b>	20133	20063
$(c_1, c_2)$	(1.0, 0.5)	1.0000, 0.5
dropout rate	0.0353	0.0387
$(P_{\text{sim}} - P_{\text{real}}) / P_{\text{real}}$	0.1553	0.1929
$(E_{\text{sim}}(X_2) - \text{mean}(x_2)) / \text{mean}(x_2)$	0.0322	0.0357
Penalty value	0.0258	0.039239
$\mathcal{E}_{X_2, y}$	0.499	0.248
$\mathcal{E}_{X_2, p_2}$	-0.016	-0.016
$\mathcal{E}_{X_2, k_2}$	0.049	0.030
$\mathcal{E}_{P, y}$	-0.732	-0.369
$\mathcal{E}_{P, p_2}$	0.029	0.028
$\mathcal{E}_{P, k_2}$	-0.224	-0.204

**Table 2:** Results based on the dataset SFSO (2006).

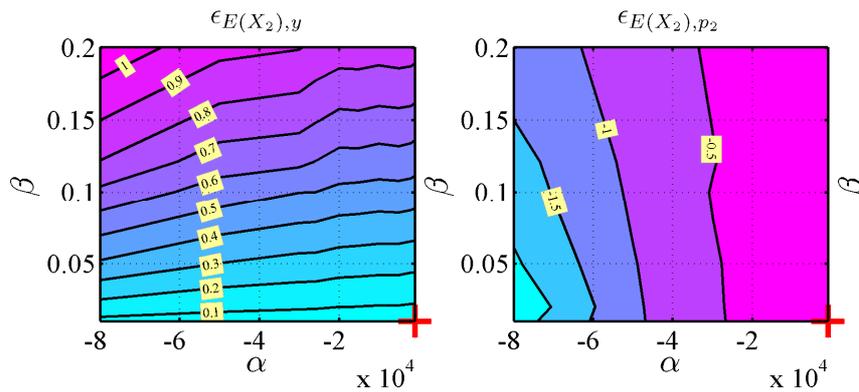
The results in the following table show, that the resulting elasticities are not realistic. The income elasticity of driving demand of  $\mathcal{E}_{X_2, y} = 0.248$  in the larger model that includes the number of persons of the household is by far lower than the value found in the international literature in the average. The same holds for the elasticity of driving demand with respect to the marginal driving costs  $\mathcal{E}_{X_2, p_2} = -0.016$  is by far too small in absolute values. The reason why both elasticities are that low in absolute value, is because both the parameters  $\alpha$  and  $\beta$  are very low. It seems in particular that  $\alpha$  is that low because for this value, the penalty function has the lowest value. The reason for that is in particular that the model does not seem to be well adaptable to the proportion of households that do not own a car since the parameter  $\alpha$  strongly affects the minimum driving distance.



**Figure 2:** Adaptation on the model, the penalty function and its components

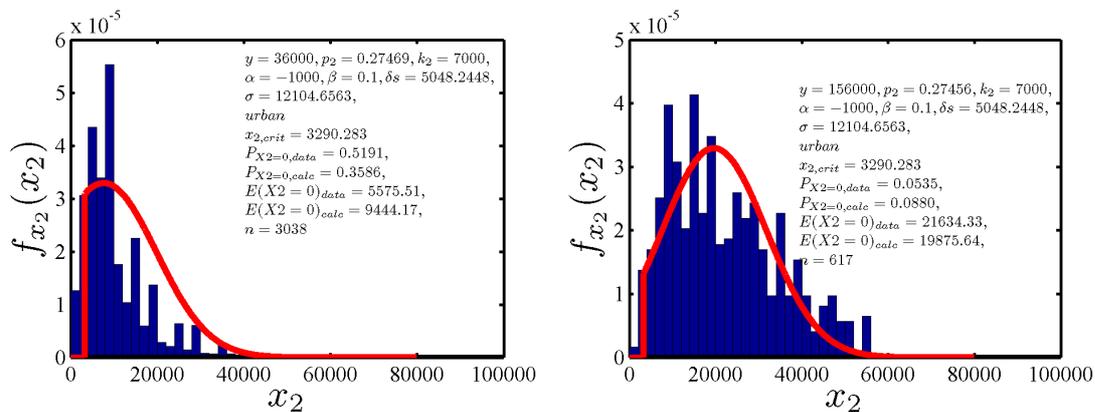
The figures below show that if the model parameter  $\alpha$  would be shifted towards larger absolute values as for instance  $\alpha = -40,000$  and  $\beta = 0.125$  so that the elasticities  $\varepsilon_{X_2, p_2}$  and  $\varepsilon_{X_2, y}$  would yield values that are in the range of what was found by other studies based on Swiss data, e.g.  $-0.30..0.67$ , respectively  $0.63..1.65$ .<sup>14</sup> But for these values the replication error with respect to the share of carless households would increase dramatically as well as the proportion of observations that would need to be eliminated from the dataset.

<sup>14</sup> See Axhausen and Erath (2010), Baranzini et. al. (2009) and Schleiniger (1995).



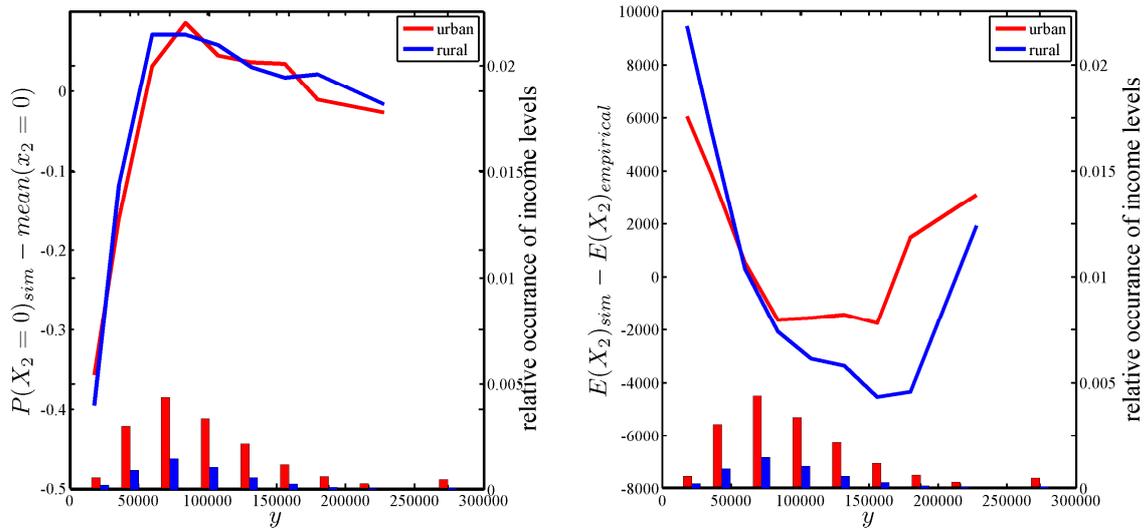
**Figure 3:** Elasticities given different parameter values (large model)

The reason for this is, that the minimum driving distance is unrealistically high for these parameter values, namely 23,200 kilometres. Another aspect of this problem is also that the probability function in the range above the minimum driving distance  $x_{2,c}$  which is determined by (2) is equal for all segments – up to a shift along the horizontal axis. This also means that the maximum density of the probability function in the range  $x_2 > x_{2,c}$  is always the same (here:  $3.3 \cdot 10^{-5}$ ). Thus, the density function cannot account for the fact that the standard deviation of  $X_2$  is smaller for lower income households, as shown in the figure below. A further problem in this case here is also that the empirical distribution of  $X_2$  is clearly positive skewed and thus the normal distribution which is symmetric might be a rather bad approximation as is also being evident when looking at the figure below.



**Figure 4:** Theoretical versus empirical distribution (small model)

The above mentioned problems also lead to forecasting errors of the probability of not owning  $P_{sim}$  a car as well as the expected value of the driving distance  $E_{sim}$ .



**Figure 5:** Forecasting errors for each household segment

For low income segments, the forecasted expectation value  $E_{sim}$  is too high, since the normal distribution of the density for the range  $x_2 > x_{2,c}$  is based on an inappropriate high  $\sigma$ -value and for the same reason  $P_{sim}$  is too small.

## 6. Conclusions and outlook

As the result above show, this model seems to be inappropriate for mapping car ownership and car use. The basic reason for that is the distribution of the driving case for the case where households drive a car does not sufficiently reflect the real data. Thus, some possible improvements need to be discussed.

The first approach I propose is quite obvious: The parameter  $\sigma$  should be small for lower incomes and large for higher income, since for higher income segments the observed driving distances vary stronger. Thus, the most simple method would be to impose a relation between the income  $y$  minus  $k_2$  and  $\sigma$ , e.g. a linear relation  $\sigma = d_0 + d_1 \cdot (y - k_2)$ . This would be equivalent of defining  $\sigma$  to be standard normal and then to change (2) to:

$$X_2 = x_2(p_2, y - k_2, \alpha, \beta, \gamma s, \varepsilon_c) = \alpha p_2 + \beta(y - k_2) + \gamma s + (d_0 + d_1 \cdot (y - k_2)) \cdot \varepsilon. \quad (9)$$

Again, this could be reformulated as:

$$X_2 = x_2(p_2, y - k_2, \alpha, \beta, \gamma s, \varepsilon_c) = \alpha p_2 + (\beta + d_1 \cdot \varepsilon)(y - k_2) + \gamma s + \sigma \varepsilon, \quad (10)$$

where  $d_0$  is now replaced by  $\sigma$  and  $\tilde{\beta} = \beta + d_1 \cdot \varepsilon$  reflects the parameter related to  $(y - k_2)$ . Since the parameter  $\beta$  has changed now, the function  $g(\cdot)$  must be changed as well. It must now depend on  $e$  that represents a realization of  $\varepsilon$ .

$$g(e) = \frac{\alpha}{\beta + d_1 \cdot e} \cdot \left( e^{\beta/\alpha(x_2(p_2, y - k_2, \alpha, \beta + d_1 \cdot e, \gamma, e) + \beta k_2)} - 1 \right) - x_2(p_2, y - k_2, \alpha, \beta + d_1 \cdot e, \gamma, e) \quad (11)$$

Consequently, the probability function is now defined as follows:

$$f(z) = \begin{cases} z = 0: \Phi\left(\frac{e_c}{d_1(y - k_2) + \sigma}\right) \\ z > x_{2,c}: \phi\left(\frac{z - \alpha p_2 - \beta y - \delta s}{d_1(y - k_2) + \sigma}\right) \end{cases}, \quad (12)$$

where  $g(e_c) = 0$   $e > -\alpha p_2 - \beta(y - k_2) - \gamma s$ . Note, that  $e$  must be in the range  $e > -\alpha p_2 - \beta(y - k_2) - \gamma s$  since this corresponds to positive solutions for  $x_2$ .

This approach seems to be very promising since it does not imply a significant additional computational effort. The limitation of that approach is that the problem that the parameters  $\alpha$  and  $\beta$  determine the minimum driving distance to a too high level<sup>15</sup> for plausible values of  $\alpha$  and  $\beta$  will be not solved

The second approach I propose is to add a fixed utility for car ownership. This would mean that the car ownership itself would provide a utility to the household. It is assumed that this utility is mainly based on the optional value of car ownership which consist on the fact that the car is always immediately available for use. This means that the indirect utility function  $V_b$  (1) is modified as follows,

$$V_b = u_b + \left( \frac{\alpha}{\beta} + \beta \cdot (y - k_2) + \alpha p_2 + \gamma s + \varepsilon \right) \cdot e^{-\beta p_2}, \quad (13)$$

where  $u_b$  denotes the fixed utility for car ownership and can be assumed to be non-negative for all households. The indirect utility function  $V_a$  (3) would remain unchanged. Thus, for any positive  $u_b$  the probability of being carless as well as the minimums driving distance would decrease, given all other parameters and economic variables remain unchanged. With this concept the question is, whether this approach would violate the assumptions of standard microeconomic theory or not. The

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<sup>15</sup> See figure A 4.2. Note that due the elasticities found in other studies the parameters  $\alpha$  and  $\beta$  are should be around 0.123 and -26,000.

reason for that is, if in this case two different utility function would be compared. So far, two values of the same utility functions but for different amounts of consumption were compared which is feasible.

The third approach I propose to choose a hedonic fixed cost of car ownership. To do so, the fixed costs will be treated as a parameter and chosen such that the criteria are optimally fulfilled. The difference between the hedonic value and the actual fixed costs are expected to be positive and can be interpreted as the monetized value of the utility of owning a car. Again, with this concept the question is, whether this approach would violate the assumptions of standard microeconomic theory or not, since the model does not any more map a true economic decision with exogenously given prices.

The fourth approach I propose is to change the functional form of the Marshallian demand function to a log-linear form. This approach was already used by De Jong (1990), but I will show, that using the concept as shown for the linear case will yield some difference compared to De Jong (1990).

## **7. Literature**

Axhausen, Kay W. and Alexander Erath, 2010, "Long term fuel price elasticity: Effects on mobility tool ownership and residential location choice", Bundesamt für Energie, 2010, Publikation 29015. <http://www.bfe.admin.ch/php/modules/enet/streamfile.php?file=000000010334.pdf&name=000000290158>

Baranzini, Andrea, David Neto and Sylvain Weber, 2009, "Élasticité-prix de la demande d'essence en Suisse", 2009, Bundesamt für Energie, 2009, Publikation 290036. [http://www.bfe.admin.ch/forschungewg/02544/02807/02808/index.html?lang=de&dossier\\_id=03910](http://www.bfe.admin.ch/forschungewg/02544/02807/02808/index.html?lang=de&dossier_id=03910)

Bhat, Chandra R., 2008, "The multiple discrete-continuous extreme value (MDCEV) model: role of utility function parameters, identification considerations, and model extensions", *Transportation Research Part B* 42 (3), 274-303.

De Jong, Gerard C., 1990, "An indirect utility model of car ownership and private car use", *European Economic Review*, Elsevier, Vol. 34(5), 971-985, July.

Dubin, Jeffrey A. and Daniel L. McFadden, 1984, "An Econometric Analysis of Residential Electric Appliance Holdings and Consumption", *Econometrica*, Vol. 52, No. 2 (Mar., 1984), 345-362.

Hausman, Jerry A., 1981, "Exact Consumer's Surplus and Deadweight Loss", *The American Economic Review*, Vol. 71, No. 4. (Sep., 1981), 662-676.

Schleiniger, Reto, 1995, "The Demand for Gasoline in Switzerland - in the Short and in the Long Run", working paper, Institute for Empirical Research in Economics 9503, 1995.

Swiss Federal Statistical Office SFSO, 2006, "Mikrozensus 2005 zum Verkehrsverhalten", Bundesamt für Statistik, Neuenburg (Switzerland) 2006.

Touring Club der Schweiz (TCS), 2007, "Kosten eines Musterautos".  
[http://www.tcs.ch/main/de/home/auto\\_moto/kosten/kilometer/musterauto.html](http://www.tcs.ch/main/de/home/auto_moto/kosten/kilometer/musterauto.html)