2013 | 2012 | 2011 | 2010 | 2009 | 2008 | 2007 | 2006 | 2005 | 2004 | 2003 | 2002 | 2001

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Rail infrastructure maintenance: a new approach

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Main sources of this presentation

- **A PSE working paper of 2010 by Gaudry and Quinet, entitled *Optimisation de l'entretien et de la régénération d'une infrastructure: exploration d'hypothèses*,**
- **A text under submission by Gaudry, Lapeyre and Quinet under the title: *Infrastructure maintenance, regeneration and service quality economics: A rail example***

Outline

- The origin of the problem
- The classical approach to rail infrastructure costs and its draw-backs
- A new approach
  - The model
  - Econometric tests
  - Consequences for infrastructure charging
- Conclusions: further research

The origin of the problem

- The sources of interest in infrastructure cost
  - Management purpose
  - Infrastructure pricing purpose
  - In the general framework of the marginal social cost pricing (MSC)

The classical approaches

- Relations between total yearly maintenance cost and several drivers, among which:
  - – Technical characteristics of the link
  - – Traffic(s) of the link
- How to assess these relations: Two approaches
  - Cost allocation through accounting or engeneering approaches
    - – well fit for average costs
  - Econometric approaches
    - – Regression between total maintenance cost and its drivers
- The most recent comprehensive work on this subject: the EU program CATRIN (2009)

A flavor of cost allocation approaches

- E.G.:The equivalence coefficients between types of vehicles

3.2.1 UK ORR's engineering model (Booz Allen Hamilton & TTCI UK 2005)

In the approach used by the ORR the sum of all variable costs estimated using the top-down approach described in the previous section is allocated to different vehicle types by use of a bottom-up engineering model. That is, cost is allocated to vehicles depending on the damage the vehicle does to the network relative to other vehicles. The distribution of costs amongst vehicle types is made according to an Equivalent Gross Tonne Mileage (EGTM) which is a weighting of the actual Gross Tonne Mileage. There are two parts to this weighting, one for damage to track (equation 1) and one for damage to structures (bridges etc., equation 2):

$$EGTM_{track} = K \; Ct \; A^{0.49} \; S^{0.64} \; USM^{0.19} \; GTM \quad (1)$$

$$EGTM_{strux} = L \; Ct \; A^{3.83} \; S^{1.52} \; GTM \quad (2)$$

where:	K	is a constant
	Ct	is 0.89 for loco hauled passenger stock and multiple units and 1 for all other vehicles
	S	is the operating speed [mph]
	A	is the axle load [tonnes]
	USM	is the unsprung mass [kg/axle]

Econometric approaches: the data

Table 13 Infrastructure characteristics, capability and condition measures used in econometric rail cost studies

Country	Great Britain	Sweden	Austria	France	Switzerland	Sweden	Finland
Study	Wheat and Smith (forthcoming)	Andersson (2006a)	Munduch et al (2002)	Gaudry and Quinet (2003)	Marti and Nschwander (2006)	Johansson and Nilsson (2002)	Johansson and Nilsson (2002)
Infrastructure characteristics	Track length Route length Length of switches	Track section distance Route length Tunnels Bridges Rail weight Rail gradient Rail cant Curvature Lubrication Joints Continuous welded rails Frost protection Switches Switch age Sleeper age Rail age Ballast age	Track section length Length of single-railed tunnels in meters Length of double-railed tunnels in meters Track radius Track gradient Length of the switches Station rails (as percentage of track length)	Number of track Apparatus Whether the track is electrified Route length Number of tracks, Automatic Traffic Control included or not	Track length Track distance (route length) Length of switches Length of Bridges Tunnels Level crossings Track Radius Track gradient Noise / fire protection Number of switches (by type) Shafts Platform edge	Track length Switches Bridges Tunnels	Track length Switches
Capability	Continuously welded rails Maximum line speed Maximum axle load	Rail weight Continuous welded rails Track quality class		Maximum line speed	Maximum line speed	Track quality index Secondary lines	Electrified Average speed
Condition	Rail age	Switch age Sleeper age Rail age Ballast age	Rail age		Rail age Sleepers age		

Source: Work carried out by Phil Wheat, ITS, University of Leeds.

Classical approach: the econometric specifications

Table 14: Methodological approaches used in econometric rail cost studies

Study	Country	Cost considered	Data type	Functional form	Number of trains/weight of trains distinction included	Input prices included
Johansson and Nilsson (2002)	Sweden	Maintenance	Panel (Pooled OLS)	Translog	✗	✗
Johansson and Nilsson (2002)	Finland	Maintenance and Maintenance plus Renewal	Panel (Pooled OLS)	Translog	✗	✗
Andersson (2006a and 2006b)	Sweden	Maintenance plus operations & Maintenance plus Operations plus Renewals	Panel (Pooled OLS and Random effects)	Translog	✓	✗
Tervonen and Idrstrom (2004)	Finland	Maintenance and maintenance plus Renewal	Panel (Pooled OLS)	First order Double Log	✗	✗
Munduch et al (2002)	Austria	Maintenance	Panel (Pooled OLS)	Double log with interaction terms	✗	✗
Gaudry and Quinet (2003)	France	Maintenance plus operations	Cross section	Unrestricted Generalized Box-Cox	✓	✗
Marti and Neuenschwander (2006)	CH	All maintenance, track maintenance plus operations, and maintenance plus renewals	Panel (Pooled OLS)	First order Double Log	✗	✗
Wheat and Smith (forthcoming)	UK	Maintenance	Cross-section	Double log with squared and cubic terms	✓	✓
Johansson and Nilsson (2002)	Sweden	Maintenance	Panel (Pooled OLS)	Translog	✗	✗

Source: Work carried out by Phil Wheat, ITS, University of Leeds.

The results

Study	Study Type	Country	Usage Elasticity	Evidence on behaviour of usage elasticity with usage	Average Marginal Cost (Euro per thousand gross tonne-km)(**)
Maintenance only					
Andersson (2006a)	Econometric	Sweden	0.204*	Falling	0.35
Wheat and Smith (forthcoming) (model IV)	Econometric	Great Britain	0.239*	Falling	1.246
Wheat and Smith (forthcoming) (model VI)	Econometric	Great Britain	0.378	Falling and then increasing	1.775
Marti and Neuenschwander (2006) Model Type 1	Econometric	Switzerland	0.200	Not tested	0.45
Marti and Neuenschwander (2006) Model Type 2	Econometric	Switzerland	0.285	Not tested	0.38
Johansson and Nilsson	Econometric	Sweden	0.1691*	Falling	0.143
Johansson and Nilsson	Econometric	Finland	0.167*	Falling	0.268
Tervonen and Idstrom (2004)	Econometric	Finland	0.133-0.175	Not tested	0.22
Munduch et al (2002)	Econometric	Austria	0.27	Not tested	0.55
Gaudry and Quinet (2003)	Econometric	France	0.37*	Increasing	Not reported
Booz Allen and Hamilton (2005)	Cost Allocation	Great Britain	0.28 for track maintenance	Not tested	1.768
Maintenance and renewals					
Andersson (2006a)	Econometric	Sweden	0.302*	Falling	0.79
Marti and Neuenschwander (2006)	Econometric	Switzerland	0.265	Not tested	0.97
Tervonen and Idstrom (2004)	Econometric	Finland	0.267-0.291	Not tested	
Booz Allen and Hamilton (2005)	Cost Allocation	Great Britain	0.19	Not tested	4.99
Renewals only					
Andersson (2006b)	Duration	Sweden	Not reported	Not tested	0.32 passenger & 0.14 freight
Booz Allen and Hamilton (2005)	Cost Allocation	Great Britain	0.19 (renewals as a whole); 0.45 for track renewals	Not tested	3.45
Operations only					
Andersson (2006a)	Econometric	Sweden	0.324	Falling then increasing	61 per train-km

(*) average elasticity. - (**) 2005/06 prices

Sources: Wheat (2007) based on Tables 6 and 7 in Lindberg (2006), and updated from Wheat and Smith (forthcoming). The studies highlighted are the latest econometric studies for maintenance and maintenance and renewal costs for each country.

Criticisms to this studies

- Does not take into account the objective of maintenance: quality of service,
  - Or assumes that the quality of service is kept constant: why?
- Rarely takes into account the fact that maintenance depends on the cumulated traffic
- Does not properly account for renewal
  - What is renewal?
    - A new system of rail, sleepers, ballast
    - Renewals takes place every 20 to 50 years, depending on the traffic and characteristics of the track
    - To be distinguished from current maintenance

The proposed model

- Principle drawn from technical analysis:
  - At the start from a renewal, quality of service is high
  - Progressively, as long as the time elapses, quality of service decreases due to traffic damages, and can be increased through current maintenance.
  - As time elapses, the maintenance level necessary to maintain quality of service is higher, as damages are linked to cumulated traffic
  - At some point of time, it is better to renew the track than to continue current maintenance

The model

- Optimization and decision variables:
  - The decision variables are current maintenance and renewal time
  - The objective function is the welfare, algebric sum of
    - (Positive): Monetary value of quality of service
    - (Negative): Current maintenance and renewal expenses
    - Discounted over the life time (infinity)
  - A new variable is introduced: Quality of service:
    - What is it: probability of break-down? Risk of speed reduction? Confort for the user?

The model: mathematical formulation

- Symbols:
  - Time :t
  - Traffic density : q(t)
  - Cumulated traffic from 0 to t: Q(t)
  - Relation between Q and q: dQ(t)/dt=q(t)
  - Technical characteristics of the link(maximal speed, number of sleepers…): K
  - Current maintenance: u(t)
  - Quality of service : S(t)
  - Successive Renewal times : T_i
  - Renewal cost: D assumed to be constant, independant of other variables
  - Discount rate: j

The model: mathematical formulation

- $dS = h(K, Q(t), t) * \left[u(t) - f(K, Q(t), q(t), t\right] * dt$

Money value of the quality of service : $q(t) * g(S(t)) = -\alpha q(t) e^{-\lambda S(t)}$

- Optimisation function:

$$M^* = \underset{u(t),T_i}{Max} \left\{ \sum_{i=0}^{i=\infty} \left[\left\{ \int_{T_i}^{T_{i+1}} [-u(t) + q(t) g(S(t))] e^{-jt} dt - De^{-jT_{i+1}} \right\} e^{-jT_i} \right] \right\}$$

Such that :

$$dS = h(K, Q(t), t) * \left[u(t) - f(K, Q(t), q(t), t\right] * dt$$

And : $0 \leq u(t) \leq m$

- Under the simplifying assumption that traffic $q(t)$ is constant over time and denoted by q, optimal regenerations will be regularly spaced at some interval T and the problem becomes:

$$M^* = \underset{u(t),T}{Max} \left\{ \left\{ \int_0^T [-u(t) - \alpha q e^{-\lambda S(t)}] e^{-jt} dt - De^{-jT} \right\} \frac{1}{1 - e^{-jT}} \right\}$$

$$= \underset{u(t),T}{Max} \left\{ [J(u(t), T) - De^{-jT}] \frac{1}{1 - e^{-jT}} \right\}.$$

The procedure

- First optimize u for a given T
- Second optimize T
- Hamiltonian:

$$H = [-u(t) - q(t) * S(t)^{-\lambda}] * e^{-jt} + y(t)\left[h(K, Q(t), t) * \left[u(t)) - f(K, Q, q(t), t)\right]\right]$$

- Pontryagin principle:

$$\underset{u}{Max}\, H = \underset{u}{Max}\left\{u(t) * \left[h(K, Q, t) * y(t) - e^{-jt}\right]\right\}$$

$$H_S + \dot{y} = 0$$

$$H_y = \frac{dS}{dt}$$

$$\underset{u}{Max}\, H = \underset{u}{Max}\left\{u(t) * \left[h(K,Q,t) * y(t) - e^{-jt}\right]\right\}$$

three possible phases:

<u>Phase A</u>: $h(K,Q,t) * y(t) - e^{-jt} < 0$. With *u(t)=0* over this interval I, we have:

- *S(t)* found by integration of $dS/dt = h(K,Q,t) * u(t) - f(K,Q,q,t)$: it is deduced, the phase starting at moment t_f with $S(t_f)$ denoting service quality at that moment;
- *y(t)* determined by $H_S + \dot{y} = 0$, which yields $y(t) = y(t_f) - \int_{t_f}^{t} \alpha q \lambda S(t)^{-\lambda-1} e^{-jv} dv$.

<u>Phase C</u>: $h(K,Q,t) * y(t) - e^{-jt} > 0$. With *u(t)=m* over this interval III, we have:

- *S(t)* by integration of $dS/dt = h(K,Q,t) * m - f(K,Q,q,t)$, the phase starting at time t_m;
- *y(t)* determined by $H_S + \dot{y} = 0$, which yields $y(t) = y(t_m) + \int_{t_m}^{t} q \lambda \alpha e^{-\lambda S} e^{-jv} dv$.

<u>Phase B</u>: $h(K,Q,t) * y(t) = e^{-jt}$. In reverse order now, we have over this central interval II:

- $y(t) = \left[1/h(K,Q,t)\right] e^{-jt}$, by simple manipulation of the phase condition;
- $S_c(t)$, determined by $H_S + \dot{y} = 0$, and to be called cruising service quality:

$$\dot{y} + H_S = \frac{-je^{-jt}}{h(K,Q,t)} - \frac{\partial h/\partial t + \left[\partial h/\partial Q\right]\left[\partial Q/\partial t\right]}{h^2(K,Q,t)} e^{-jt} + \alpha q \lambda S_c(t)^{-\lambda-1-jt} = 0,$$

(8) $$Log[S_c(t)] = \frac{1}{\lambda+1}\left\{Log(\sum_{i=1}^{i=c}\alpha_i q_i) + Log(\frac{\lambda}{j}) - Log\left[\frac{1}{h} + \frac{h'}{jh^2})\right]\right\},$$

with *u(t)* then given by:

(9) $$u(t) = f(K,Q,q,t) + \frac{1}{h(K,Q,t)}\frac{dS_c(t)}{dt}.$$

For system behavior, and starting for convenience at T, the end of the period, transversality condition $y(T)=0$ then requires to be in Phase A, and current maintenance $u(t)$ to be *nil*. Two possibilities arise when one starts backing-up in time:

(i) either one stays in Phase A because $y(t)$ satisfies the corresponding inequality restriction. The resulting optimal policy is then to perform no current maintenance and to regenerate periodically.

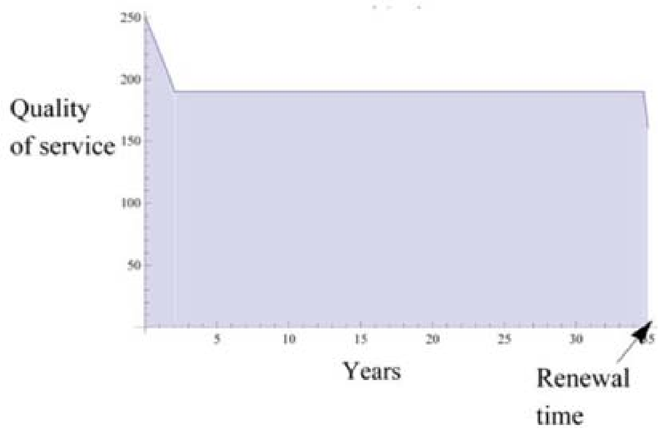
(ii) or, at a certain instant t_f, one has $y(t_f)=\left[1/h(K,Q(t_f),t_f)\right]e^{-jt_f}$, and service quality then follows trajectory $S_c(t)$. Further, as one approaches period beginning $t=0$, a number of possibilities arise depending on how service quality $S(0)$ achieved by the previous regeneration compares to $S_c(0)$.

If, as in standard practice, $S(0)>S_c(0)$, one again reaches a Phase A state, with $u(t)=0$, *i.e.* devoid of maintenance.

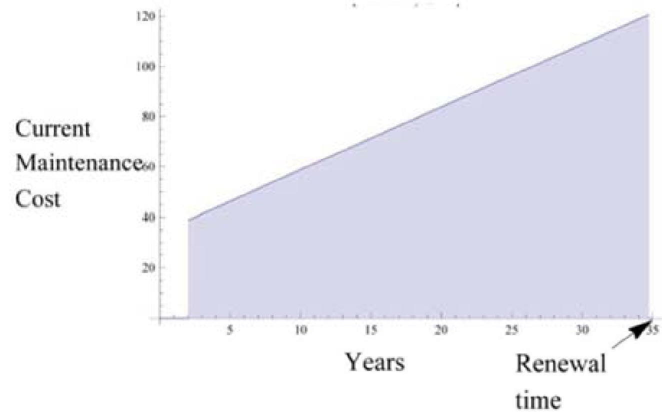
Typical evolutions

Typical evolution of service quality and maintenance expense for T=35

A. Service quality (initial value equals 250)



B. Maintenance expenditure (*nil* close to T=35)



Determination of the optimal renewal horizon T

The assumption of stationary traffic, made to some extent for convenience in the previous section, is now required for an easy resolution. If *J*(T)* is, for given *T*, the highest value of *J* solving $\underset{u}{Max}\{J(u(t),T)\} = J^*(T)$, the optimal duration *T* is that which maximizes

(13) $$M(T) = \frac{J^*(T) - De^{-jT}}{1 - e^{-jT}} .$$

Introducting uncertainty

(21) $$dS(t) = h(K,Q,t)[u(t) - f(K,Q,q,t)]dt + \sigma dz$$

where the random variable *dz* denotes a classical Brownian motion, as in Haussmann & Suo (1995a). We will not analytically solve this optimisation problem but simply perform numerical simulations using the usual Hamilton-Jacobi-Bellman (HJB) equation with partial derivatives:

(22) $$-J_t = \underset{u}{Max}\{[-u(t) - \alpha q e^{-\lambda S(t)}]e^{-jt} + J_S\left[h(K,Q,t)u(t) - f(K,Q,q,t)\right] + \sigma^2 J_{SS}\}.$$

Simulations on uncertainty

- Under uncertainty, the quality of service is a target:
  - Fully reached in continuous time
  - Reached with corrections in (real) discrete time

Parameters and functions used for the simulations

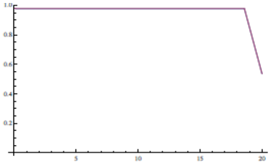
Number of years between 2 renewals: $T=20$.	Initial quality of service: $S(0)=3,3$.
Limit values of current maintenance $u(t)$: 0; 0,2.	Yearly traffic normalized to: $q=100$.
Discount rate: $j=0,04$.	
Functions for: $dS(t) = h(K,Q,t)[u(t) - f(K,Q,q,t)]dt + \sigma dz$:	$h(t)=7$; $f(q,t)=[(0,25+0,16(q/400)^2)*(1+0,04(q/400)t)]/[7e^{-0,02}]$.
Standard deviation of Wiener random variable: $\sigma=0$; then $\sigma=0,1$; then $\sigma=0,2$. [the measured standard deviation of the observed quality of service is approximately 0,08].	
Parameters of the function expressing the value of the quality of service $-\alpha e^{-\lambda S}$: $\alpha=10$; $\lambda=10$.	
Note that technical parameter values denoted by K earlier are taken into account in the values selected.	

Figure 1. Analytically derived Pontryagin and Hamilton-Jacobi-Bellman simulation results

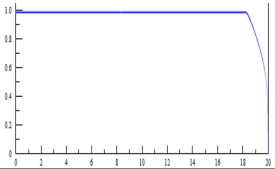
Case	A. Analytical Pontryagin result	B. HJB simulation σ=0	C. HJB simulation σ=0,1	D. HJB simulation σ=0,2
Target service quality S	0,976996	0,98	0,97	0,96
J integral	-0,256568	-0.259 +/-0.0015	-0.261 +/-0.003	-0.263 +/-0.005
Marginal cost: dJ/dq	0,000919	0,0006 +/-0,0002	0,0006 +/-0,00044	0,0010 +/-0,0007

TARGET TRAJECTORIES

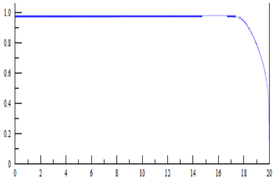
A. Analytical Pontryagin result



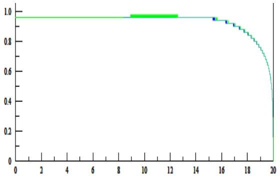
B. HJB simulation σ=0



C. HJB simulation σ=0,1



D. HJB simulation σ=0,2



Econometric calibrations

Three Calibrations

The Target Service relationship::

(27-A) $$\ln[S_c(t)] = \frac{1}{\lambda+1}\left\{\ln(\sum_{i=1}^{i=c}\alpha_i q_i) + \ln(\alpha\lambda) - \ln(h) + \ln\left[j - \frac{h'}{h})\right]\right\}.$$

The current Maintenance relationship.

(23-C) $$u_t = f(K,Q,q,t)_t + \frac{1}{h(K,Q_t,t)_t}\left[Sc_t - Sc_{t-1}\right]_t + a(K,Q,t)_t\left[S_{t-1} - Sc_{t-1}\right]_t + \varepsilon_{1t},$$

The technical relation linking quality of service, maintenance and traffic:.

(25) $$S_t - S_{t-1} = h(K,Q_t,t)_t \cdot \left[u_t - f(K,Q_t,q,t)_t\right] + \varepsilon_{2t},$$

which, after replacement of $u_t - f(K,Q_t,q,t)_t$ by its value from (23-C), may be written:

(26) $$S_t - S_{t-1} = Sc_t - Sc_{t-1} + \frac{a(K,Q,t)_t}{h(K,Q,t)_t}\{(S_{t-1} - Sc_{t-1}) + [\varepsilon_{1t}]\} + \varepsilon_{2t},$$

where [$\varepsilon_{1t} = u_t - E(u_t)$] can be estimated from (23-C)

The available data

Table 1. Mean values of principal variables available by track segment

	Variables	1999 (sample size 985)			2007 (sample size 700)		
u_t	**Current maintenance expenses**						
	total cost per km (current Euros)	68 899			52 432		
	surveillance	--			*14 821*		
	maintenance	--			*37 611*		
e_0	**Technical state variables K**						
	length of segment (meters)	18 991			28 219		
	length of all tracks (meters)	by number of tracks			46 186		
	electrified or not; tension (1,5 or 25 kV)[1]	yes			yes		
	number of switches per segment	21,41			25,48		
h_0	**Initial standing**						
	maximum allowed speed (km/h)	127,77			114,69		
	UIC group classification	(reconstructed)			observed		
	high speed rail line	yes			yes		
	suburban line	yes			yes		
h_t	**Current condition**						
	average age of rails (years)	26,59			30,56		
	average age of sleepers (years)	26,87			28,07		
S_t	**Service quality of the track**						
	NL index of longitudinal track rectitude (mm)	--			1,451[2]		
q_t	**Traffic per day: trains and gross tons**	**trains**	**weight**	***(per train)***	**trains[3]**	**weight[3]**	***(per train)***
	GL : long distance passenger trains (VFE)	6,04	3 151	*(522)*	17,99	10 255	*(570)*
	TGV: high speed trains	----	----	----	*9,25*	*5 612*	*(601)*
	Classic intercity trains (Corail)	----	----	----	*8,74*	*4 643*	*(531)*
	TER : regional passenger trains	5,09	1 115	*(219)*	24,44	5 559	*(228)*
	IdF : Île-de-France passenger trains	5,99	2 378	*(397)*	16,21	6 215	*(384)*
	Fret : freight trains	6,10	6 403	*(1049)*	13,70	15 484	*(1131)*
	HLP : locomotives	1,40	138	*(99)*	2,84	275	*(97)*
	Total for the six categories of trains	24,76	13 185	*(533)*	75,16	37 788	*(503)*

[1]Out of a total network of about 30 000 km, 11 582 km (including 1 884 km of TGV lines) have alternating tension of 25 kV and 5 863 km have continuous tension of 1,5 kV; 126 km are electrified otherwise (third rail, *etc.*). [2]Available over the period 2000-2010 for a subset of 608 observations. [3]Available over the period 1995-2007 for all 700 observations.

The Box-Cox transform

$$\frac{Y^{\lambda_Y} - 1}{\lambda_Y} = \sum \beta_{X_k} \frac{X_k^{\lambda_{X_k}} - 1}{\lambda_{X_k}} + \sum \alpha_i Z_i$$

The Target Service relationship::

$$\ln[S_c(t)] = \frac{1}{\lambda+1}\left\{\ln(\sum_{i=1}^{i=c}\alpha_i q_i) + \ln(\alpha\lambda) - \ln(h) + \ln\left[j - \frac{h'}{h})\right]\right\}. \quad (27\text{-A})$$

Table 1. Embedded Target Service S_c function results (4060 obs., 2001-2007)

Elasticity $\eta(S)$, λ, *t*-statistic* of β_k coefficient			$\eta(S)$	λ
β_0	Intercept	(*t*=0)	n.a. (176.66)	
n	Total number of trains	(*t*=0)	0.020 (30.22)	**0**
	Long distance GL train share (ref.: TGV)	(*t*=0)	-0.007 (-4.85)	
	Regional TER train share (ref.: TGV)	(*t*=0)	-0.006 (-7.80)	
	Ile-de-France regional train share (ref.: TGV)	(*t*=0)	-0.001 (-4.15)	
	Freight train share (ref.: TGV)	(*t*=0)	-0.008 (-7.43)	
	Locomotive only HLP train share (ref.: TGV)	(*t*=0)	-0.000 (-0.66)	
$\mathbf{e_0}$	Segment length	(*t*=0)	0.004 (4.12)	**0**
	Track length	(*t*=0)	0.018 (6.75)	**0**
	Electrified 1500 V (ref.: not electrified)	(*t*=0)	0.007 (3.17)	
	Electrified 25000 V (ref.: not electrified)	(*t*=0)	0.002 (1.05)	
	Number of switches	(*t*=0)	-0.002 (-3.34)	**1**
$\mathbf{h_0}$	Maximum allowed speed	(*t*=0)	0.053 (26.83)	**0**
	Suburban line (ref.: other line)	(*t*=0)	-0.005 (-1.52)	
	High speed rail line (ref.: classic line)	(*t*=0)	-0.019 (-3.70)	
W	Cumulative total tons per km of track	(*t*=0) [*t*=1]	-0.001 (-2.83)	**1.41** (5.11) [1.48]
t	Time since last regeneration (age of rails)	(*t*=0) [*t*=1]	-0.028 (-17.84)	**2.09** (7.83) [4.09]
w	Current tons per km of track	(*t*=0)	0.001 (1.06)	**1**
β_d	6 yearly dummies (ref.: 2001)			
	Log likelihood		**-1384.11**	

The current Maintenance relationship.

$$(23\text{-C}) \qquad u_t = f(K,Q,q,t)_t + \frac{1}{h(K,Q_t,t)_t}\left[Sc_t - Sc_{t-1}\right]_t + a(K,Q,t)_t\left[S_{t-1} - Sc_{t-1}\right]_t + \varepsilon_{1t} \, ,$$

Table 1. Phase B Maintenance cost *u* without max. speed *v* (580 obs., 2007)

Elasticity η(u), λ , (*t*-stat.=0)*, [*t*-stat.=1]		η(u)	λ
u	Total maintenance cost per km (dependent variable) (*t*=0) [*t*=1]	n.a.	**0,22** (17.40) [-61.08]
β_0	Intercept (*t*=0)	n.a. (10.17)	
S	Target Δ service : 2006-2005 $[E(S_{ct-1}) - E(S_{ct-2})]$ (*t*=0)	0.10** (5.18)	**1**
	Trajectory correction: (obs.-target)$_{2005}$ $[S_{t-2} - E(S_{ct-2})]$ (*t*=0)	-0.002 (-3.63)	
e_0	Segment length (*t*=0)	-0.028 (-0.75)	**0.50** (8.37) [-8.34]
	Track length (*t*=0)	0.174 (2.26)	
	Number of switches (*t*=0)	0.434 (19.18)	
W+w	Cumulative+ current total tons (W+w) (*t*=0) [*t*=1]	0.247 (9.06)	**0.39** (2.42) [-3.81]
t	Time since last regeneration (agerail) (*t*=0)	0.043 (0.56)	**0**
R_1	Same region: ρ_1 (*t*=0)	*0.589* *(6.87)*	
	Log likelihood	**-6825.77**	
	Number of β_k estimated	8	
	Number of λ_k estimated	2	
	Difference in degrees of freedom	6	
	Variant run number	105	

The technical relation linking quality of service, maintenance and traffic:.

(25) $$S_t - S_{t-1} = h(K,Q_t,t)_t \cdot \left[u_t - f(K,Q_t,q,t)_t\right] + \varepsilon_{2t},$$

which, after replacement of $u_t - f(K,Q_t,q,t)_t$ by its value from (23-C), may be written:

(26) $$S_t - S_{t-1} = Sc_t - Sc_{t-1} + \frac{a(K,Q,t)_t}{h(K,Q,t)_t}\{(S_{t-1} - Sc_{t-1}) + [\varepsilon_{1t}]\} + \varepsilon_{2t},$$

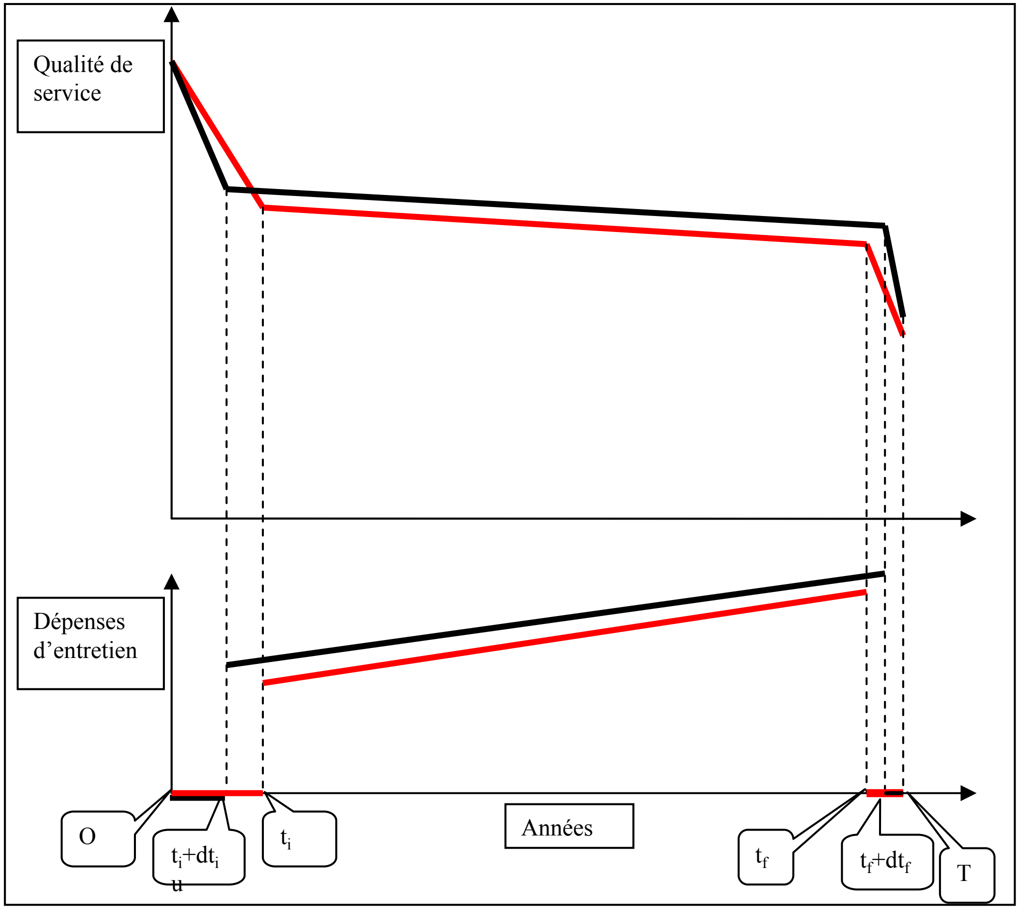
where [$\varepsilon_{1t} = u_t - E(u_t)$] can be estimated from (23-C)

Table 1. Explaining Service quality changes ΔS by factors defined for 2007

	Linear model: dependent variable ΔS_t:	2007-2006
	Column	3
	Elasticity η(ΔS) and *t*-statistic of β_k	**η(ΔS)**
β_0	Intercept (*t*=0)	n.a. (3.76)
$E(S_{ct})$- $E(S_{ct-1})$ (t=2007)	Δ Target Service (2007-2006) (*t*=0)	0.209 (0.94)
S_{t-1}- $E(S_{ct-1})$ (t=2007)	Trajectory Corr. (obs.-target)$_{2006}$ (*t*=0)	-0.027 (-4.50)
	Maintenance cost Surprise 2007 based on Column 4.A run of Table 13 (*t*=0)	0.003 (0.53)
	Log likelihood	**205.79**
	Number of β_k estimated	4
	Variant run number	15

Consequences for marginal social cost pricing

- At a given time



Consequences for marginal social cost pricing

- The effect of quality of service is not neglegible
- A kind of Morhing effect for small traffics flows

Table 1. Revenue differences between standard and optimal intertemporal pricing rules

Traffic	Revenue from standard marginal charge	Revenue from new optimal charge
400	1,509	1,630
300	1.080	1.140
200	0,583	0,588
100	0,173	0,155
50	0,057	0,038

Conclusions

- A model linking
  - current maintenance and renewal
  - Infrastructure expenses and quality of service
- Conclusions in accordance with technical experience
- A good match between theory and statistical evidence:
  - Does the operator optimize its behaviour?
- Possible extensions:
  - Non constant traffic flow
  - Optimal timing of renewal in presence of uncertainty
  - Better data on quality of service, on cumulated traffics
  - Other specifications for the damage law
  - Valuation of quality of service