Macroscopic Traffic Control of a Mixed Urban and Freeway Network

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Abstract: In this paper, the macroscopic traffic control of a large-scale mixed transportation network consisting of freeway and urban network is tackled. The urban network is partitioned in two regions, each one with a well-defined macroscopic fundamental diagram (MFD), i.e. a unimodal and low-scatter relationship between network density and outflow. The freeway is regarded as one alternative commuting route which has one on-ramp and one off-ramp within each urban region. The urban and freeway flow dynamics are formulated with the tool of MFD and asymmetric cell transmission models, respectively. Four controllers are considered to control the flow distribution between urban regions and freeway: (i) two on the border of urban regions operating to manipulate the perimeter interflow rates between the two regions, and (ii) two other controllers on the on-ramps for ramp metering to control the flow rates from urban roads to the freeway. The optimal traffic control problem for the mixed network is solved by a receding horizon approach in order to maximize the number of trips that reach their destinations. The results of this paper can be extended to develop efficient control strategies for large-scale mixed traffic networks.

Keywords: Traffic control, Optimal control, Model-based control, Optimization problems, State feedback, Dynamic models.

1. INTRODUCTION

Metropolitan transportation networks have a hierarchical structure which essentially comprise of freeways and urban roads providing the interrelated infrastructure for mobility and accessibility. The freeway and the urban network are inherently coupled yet have dissimilar traffic flow dynamics which challenge the traffic control problem for mixed networks.

Traffic networks in large cities are a mixture of two traffic control entities: urban network and freeways. Integrating the two entities during heavy congestion conditions through efficient mixed control policy will provide efficient performances compared with separate control policies. An efficient control policy would control and manage the flow distribution between the urban network and the freeway, e.g. drivers can travel from the periphery to the city center by choosing between two routes: either traveling through the freeway or the urban network. The mixed control policy can affect the route choice, thereby the flow distribution, to optimize the whole traffic network.

Ramp metering is the most commonly used controller in freeways. The ramp metering controller manipulates the flow rate entering the freeway from the urban roads surrounding it. Local and coordinated control strategies were proposed and implemented for ramp metering. In local control strategies, the control policy for an on-ramp is determined according to the traffic condition in the surrounding region including the downstream of the freeway and the upstream of the on-ramp. ALINEA controller, proposed in Papageorgiou et al. (1991), is an example of local feedback control. It is a P-controller where the inflow rate to the freeway is determined corresponding to the "error" which is the difference between the downstream density and the reference or the desired density (defined in advance). In coordinated strategies, the control policy for multiple on-ramps are determined according to the traffic conditions in multiple regions including number of on-ramps and areas in the freeway. The coordinated ramp metering is in fact a multi-regulator controller as all on-ramps metering attempt to operate the freeway traffic conditions near the desired densities. Overviews of local and coordinated ramp metering control are presented in Papageorgiou and Kotsialos (2002); Lipp et al. (1991); Geroliminis et al. (2011). Ramp metering might not efficiently operate in case of downstream bottleneck restrictions, for example a high demand off-ramp queue spillbacks in the freeway and blocks a mainline line. Also, in case a freeway ends inside a congested city center, ramp metering might not be able to increase the outflow. In these cases arterial and freeway networks should be controlled in an integrated matter.

In urban networks, the macroscopic fundamental diagram (MFD) aims to simplify the micro-modeling task of the urban network where the collective traffic flow dynamics of subnetworks capture the main characteristics of traffic congestion, such as the evolution of space-mean flows and densities in different regions of the city. The MFD of urban traffic provides for different network regions a unimodal, low-scatter relationship between network vehicle density [veh/km] and network space-mean flow or outflow [veh/hr] if congestion is roughly homogeneous in the region. Alternatively, the MFD links *accumulation*, defined as the number of vehicles in the region, and *trip completion flow*, defined as the output flow of the region. Network flow or trip completion flow increases with accumulation up to a critical point, while additional vehicles in the network cause strong reductions in the flow. The physical model of MFD was initially proposed by Godfrey (1969) and observed with dynamic features in congested urban networks in Yokohama by Geroliminis and Daganzo (2008), and investigated using empirical or simulated data by Buisson and Ladier (2009); Ji et al. (2010); Mazloumian et al. (2010); Daganzo et al. (2011) and others. A solution for heterogeneous networks is that they can be partitioned to a number of homogeneous regions with small variances of link densities so that region will have a well-defined MFD, see Ji and Geroliminis (2011).

Recently, Haddad et al. (2012) utilized the MFD to introduce elegant perimeter control to improve mobility and decrease delays in large urban networks.

The results encourage us to utilize the MFD and its perimeter controls for the mixed urban and freeway network. In this paper, the MFD is used to model the urban network and the perimeter controllers are integrated with the ramp metering controllers. The urban network is assumed to be partitioned into two urban regions having their own MFDs, while the freeway passes through both urban regions having one on-ramp and one off-ramp within each region. The traffic dynamics of the freeway are modeled according to the asymmetric cell transmission model (ACTM) in Gomes and Horowitz (2006). Recent study with empirical data (Geroliminis and Sun (2011)) has shown that a freeway system might not be well-described with an MFD because of strong hysteresis phenomena.

In this paper, the optimal control problem for a mixed network is formulated. The optimal policy aims to maximize the trip completion in the whole network by manipulating (1) the inflows to the freeway from the urban network through the ramp metering controllers, and (2) the flows transfer between urban regions through the perimeter controllers. The optimal control problem is solved by a receding horizon framework, and the results are compared with a local feedback controller for a few case study examples.

2. A MIXED URBAN AND FREEWAY NETWORK

This section describes the mixed network. Let us consider a mixed urban and freeway network as shown in Fig. 1. The traffic dynamics of an MFD, linking space-mean flow, density, and speed of a large urban area, is utilized to model the urban network. It is assumed that the urban network is heterogeneous and partitioned into two homogeneous urban regions, denoted by (1) and (2), having their own MFDs. While there is a freeway, denoted by (3), that passes through both urban regions having one on-ramp and one off-ramp within each region. The traffic dynamic of the freeway is modeled according to the asymmetric cell transmission model (ACTM) described in Gomes and Horowitz (2006), which is an extension version of the cell transmission model in Daganzo (1994).

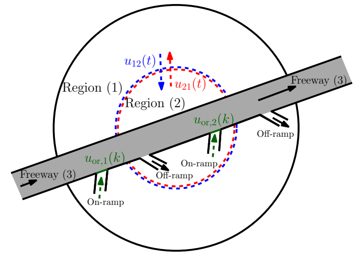

Fig. 1. A mixed urban and freeway network: two regions (1) and (2), and a freeway (3).

To keep elegant the dynamics formulation, the following assumptions are made regarding the trip routes in the mixed network: (A1) the freeway can be used at most once during the trip (exit and re-enter is not allowed), (A2) there is at most one urban region transfer during the trip, e.g. traveling from 1 to 2 then to 1 is not allowed. Under these assumptions, an origin-destination trip might have at most two routes since the traveler can choose between using the urban network or the freeway, or using both the urban network and the freeway (if this option exists), e.g. there are two routes from 1 to 1: $1 \to 1$ traveling from 1 to 1 using the urban network of 1, and $1 \to 3 \to 1$ traveling from 1 to 1 using first the urban network of 1 and then the freeway. All origin-destination trip routes in the mixed network are summarized in Table 1.

Table 1. Trip routes and demands in the mixed network.

O\ D	1	2	3
1	$q_{11} : 1 \to 1$ $q_{131} : 1 \to 3 \to 1$	$q_{12} : 1 \to 2$ $q_{132} : 1 \to 3 \to 2$	$q_{13} : 1 \to 3$ $q_{123} : 1 \to 2 \to 3$
2	$q_{21} : 2 \to 1$ $q_{231} : 2 \to 3 \to 1$	$q_{22} : 2 \to 2$	$q_{23} : 2 \to 3$ $q_{213} : 2 \to 1 \to 3$
3	$q_{31} : 3 \to 1$ $q_{321} : 3 \to 2 \to 1$	$q_{32} : 3 \to 2$ $q_{312} : 3 \to 1 \to 2$	$q_{33} : 3 \to 3$

Let $Q_{ij}(t)$ [veh/sec] be the total demand generated in origin i with destination to j at time t, $i, j = 1, 2, 3$. We distinguish between two demands for the same origin-destination: $q_{ij}(t)$ [veh/sec] denotes a generated demand in origin i with destination j at time t that belongs to the trip route $i \to j$, while q_{ikj} [veh/sec], $k \neq i, j$, denotes a generated demand in origin i with destination j at time t that belongs to the trip route $i \to k \to j$. All origin-destination demands are also summarized in Table 1. Note that $Q_{ij}(t)$ is the sum of $q_{ij}(t)$ and $q_{ikj}(t)$ (if it exists). Let θ $[-]$ be a priori known ratio, $\theta \in [0, 1]$, where $q_{ij}(t) = \theta \cdot Q_{ij}(t)$ and $q_{ikj}(t) = (1 - \theta) \cdot Q_{ij}(t)$ if $q_{ijk}(t)$ exists, otherwise $q_{ij}(t) = Q_{ij}(t)$. e.g. $Q_{11}(t) = q_{11}(t) + q_{131}(t)$, $q_{11}(t) = \theta \cdot Q_{11}$, $q_{131}(t) = (1 - \theta) \cdot Q_{11}$, while $Q_{22}(t) = q_{22}(t)$. In this paper, due to sized limitations, the value of parameter θ is constant (with time and O-D type) input parameter to the model. In the more sophisticated approach (ongoing work) θ's will be embedded in the modeling formulation and traffic conditions in the different alternatives will dynamically specify their values.

Corresponding to the above traffic demands, six accumulation states are used to model the dynamic equations of *the urban network*: $n_{ij}(t)$ [veh], $i = 1, 2; j = 1, 2, 3$, where $n_{ij}(t)$ is the total number of vehicles in region i with next destination j at time t. Let us denote $n_i(t)$ [veh] as the accumulation or the total number of vehicles in region i at time t, i.e. $n_i(t) = \sum_{j=1}^{3} n_{ij}(t)$.

The MFD is defined by $G_i(n_i(t))$ [veh/sec] which is the trip completion flow for region i at $n_i(t)$. The trip completion flow for region i is the sum of transfer flows, i.e. trips from i with destination j, $i \neq j$, plus the internal flow, i.e. trips from i with destination i. The transfer flow from i to j is calculated corresponding to the ratio between accumulations, i.e. $M_{ij} = (n_{ij}/n_i) \cdot G_i(n_i(t))$, $i \neq j$, while the internal flow from i with destination to i is calculated by $M_{ii} = (n_{ii}/n_i) \cdot G_i(n_i(t))$. These relationships assume that trip length for all trips within a region (internal or external) are similar. For a description in different case the reader can refer to Geroliminis (2009), which will not alter the methodology. Simulation and empirical results, Geroliminis and Daganzo (2008), show that the shape of MFD can be approximated by a non-symmetric unimodal curve skewed to the right, i.e. critical density that maximizes network flow is smaller than half of jammed density. Thus, we utilize a 3rd polynomial function of $n_i(t)$, e.g. $G_i(n_i(t)) = a_i \cdot n_i^3 + b_i \cdot n_i^2 + c_i \cdot n_i$, where a_i, b_i, c_i are estimated parameters, e.g. from real data.

2.1 Modeling the freeway in the mixed network

The traffic dynamics of *the freeway* in the mixed network are based on the ACTM. The mass conservation equations of the on-ramps are adjusted to fit the mixed network problem as the input demands of the on-ramps are in fact the output of the MFDs. Moreover, the off-ramp flows of the freeway are the input demands for the urban network, therefore, new equations are formulated to split the off-ramp flows to different origin-destination demands for the MFDs.

In the following, a brief description of the ACTM is presented, while the reader can refer to Gomes and Horowitz (2006); Daganzo (1994) for a full description. The ACTM divides the freeway to L "cells", where each cell l of the freeway contain at most one on- or one off-ramp. Three cells of the freeway are schematically shown in Fig. 2 as cell l has an on-ramp belonging to region i. The number of vehicles in cell l at time step k, $k = 0, 1, \ldots, K-1$, is denoted by $x_l(k)$ [veh], while $f_l(k)$ [veh] is the number of vehicles moving from cell l to $l+1$ during time step k. The on-ramp is feeded from $M_{i3}(t)$, the demand generated in region i of the urban network with destination to the freeway, calculated by the MFD. Let $n_{\text{or},i}(k)$ [veh] be the queue length of the on-ramp in region i at time step k, and $n_{\text{or},i,\max}$ [veh] be the maximum queue size of the on-ramp in region i. It is assumed that each cell l has a triangular fundamental diagram with the following parameters: $w_l \in [0, 1]$ is the normalized congestion wave speed, $v_l \in [0, 1]$ is the normalized free-flow speed, $\bar{x}_l$ [veh/lane] is the jam accumulation, and $\bar{f}_l$ [veh/hour/lane] is the mainline capacity. The freeway network is integrated to the urban network through the following equations.

The unmetered on-ramp flow $f_{\text{or},l}(k)$ [veh] is the number of vehicles that can enter cell l from its on-ramp during time step k. It is calculated as follows

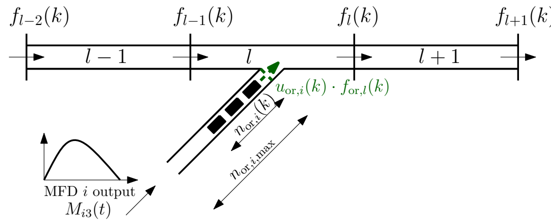

Fig. 2. Three cells of the freeway in the ACTM. Cell l has an on-ramp belonging to region i.

$$f_{\text{or},l}(k) = \min\big[n_{\text{or},i}(k) + M_{i3}(t) \cdot T_k, \; , \xi_l \cdot (\bar{x}_l - x_l(k)), \; s_{\text{or},i}(k)\big] \tag{1}$$

where i is the region that the on-ramp belongs to, T_k [sec] is the time step size, $s_{\text{or},i}(k)$ [veh] is the maximum number of vehicles that can enter the freeway in saturated conditions for the on-ramp belonging to region i at time step k, and ξ_l $[-] \in [0, 1]$ is the on-ramp flow allocation parameter, see Gomes and Horowitz (2006).

The on-ramp metering controllers, denoted by $u_{\text{or},i}(k)$ $[-]$, $i = 1, 2$, are introduced on the entrance of the freeway in region i, see Fig. 1. The on-ramp controllers meter the flow entering the freeway. The queue dynamics for the on-ramp belonging to region i with ramp metering controller $u_{\text{or},i}(k)$ is as follows

$$n_{\text{or},i}(k+1) = \min\big(n_{\text{or},i}(k) + M_{i3}(t) \cdot T_k - \; - u_{\text{or},i}(k) \cdot f_{\text{or},l}(k), \; n_{\text{or},i,\max}\big) \tag{2}$$

The mainline flow in the freeway is calculated as follows:

$$f_l(k) = \min\big[(1 - \beta_l(k)) \cdot v_l \cdot (x_l(k) + \gamma \cdot f_{\text{or},l}(k)), \; , F_l(k), \; w_{l+1} \cdot (\bar{x}_{l+1} - x_{l+1}(k) - \gamma \cdot f_{\text{or},l+1}(k))\big] \tag{3}$$

where $F_l(k) \triangleq \min\left\{\bar{f}_l, \; \frac{1-\beta_l(k)}{\beta_l(k)} \cdot \bar{f}_{\text{off},l}\right\}$, $\bar{f}_{\text{off},l}$ [veh] is the off-ramp capacity, γ $[-] \in [0, 1]$ is the on-ramp flow blending coefficient, and $\beta_l(k)$ $[-]$ is the split ratio for the off-ramp in cell l. The exit flow of the off-ramp in cell l, $f_{\text{off},l}(k)$ [veh], is calculated as follows:

$$f_{\text{off},l}(k) = \frac{\beta_l(k)}{1 - \beta_l(k)} \cdot f_l(k) \tag{4}$$

The mainline mass conservation is

$$x_l(k+1) = x_l(k) + f_{l-1}(k) + u_{\text{or},i}(k) \cdot f_{\text{or},l}(k) - \; - f_l(k) - f_{\text{off},l}(k) \tag{5}$$

for $l = 1, 2, \ldots, L$, and $k = 0, 1, \ldots, K-1$, where $f_{\text{or},l}(k) = 0$ and/or $f_{\text{off},l}(k) = 0$ if cell l does not contain on-ramp and/or off-ramp, respectively.

For each cell of the freeway, we do not keep track of the origins and destinations of vehicles. But, vehicles existing from the off-ramp can have multiple destinations. To calculate the off-ramp flow distribution we assume that the exit ratios are similar to the O-D table ratios. Hence, in order to integrate this effect the off-ramp exit flows, denoted by "hat" variables, for off-ramp of region 1 are:

$$[\hat{q}_{31}(t), \hat{q}_{312}(t), \hat{q}_{231}(t)] = R_1 \cdot [q_{31} + q_{131}, q_{312}, q_{231}] \tag{6}$$

where

$$R_1 = \frac{f_{\text{off},1}(k)}{T_k} \cdot \frac{1}{q_{31} + q_{131} + q_{312} + q_{231}} \tag{7}$$

and similar for off-ramp of region 2.

3. THE MIXED NETWORK CONTROL PROBLEM

3.1 Problem formulation

In the mixed network control problem, there are two types of controllers: the perimeter controllers of the urban regions and the on-ramp metering control of the freeway. The goal of controlling the mixed network is to maximize the total number of vehicles that complete their trips and reach their destinations. The perimeter controllers denoted by $u_{12}(t)$ and $u_{21}(t)$ $[-]$ are introduced on the border between the two regions as shown in Fig. 1, where the purpose is to control the transfer flows. Since the perimeter controllers exist only on the border between the two regions, the internal flows cannot be controlled or restricted, while the transfer flows are controlled by the controllers such that only a ratio transfers at time t. The perimeter controllers $u_{12}(t)$ and $u_{21}(t)$, where $0 \leq u_{12}(t), u_{21}(t) \leq 1$, are the ratio of the transfer flow that transfers from region 1 to 2 and region 2 to 1 at time t, respectively. The on-ramp metering controllers $u_{\mathrm{or},i}(k)$, $i = 1, 2$, are introduced on the entrance of the freeway and meter the flow entering the freeway. The control problem has six state variables describing the dynamics of the urban network, two state variables describing the queue dynamics of the freeway on-ramps, and L states of accumulation for the cells of the freeway. Therefore, the mixed control problem is formulated as follows:

$$J = \max_{\substack{u_{12}(t),\, u_{21}(t),\\ u_{\mathrm{or},1}(k),\, u_{\mathrm{or},2}(k);\\ \text{for } k=0,\ldots,K-1}} \int_{t_0}^{t_\mathrm{f}} \Big[M_{11}(t) + M_{22}(t) \Big] \mathrm{d}t + \sum_{k=0}^{K-1} f_L(k) \quad (8)$$

subject to

$$\frac{\mathrm{d}n_{11}(t)}{\mathrm{d}t} = \frac{\hat{q}_{321}(t) + q_{21}(t)}{\hat{q}_{321}(t) + q_{213}(t) + q_{21}(t)} \cdot u_{21}(t) \cdot M_{21}(t) + q_{11}(t) + \hat{q}_{231}(t) + \hat{q}_{31}(t) - M_{11}(t) \quad (9)$$

$$\frac{\mathrm{d}n_{12}(t)}{\mathrm{d}t} = q_{12}(t) + q_{123}(t) + \hat{q}_{312}(t) - u_{12}(t) \cdot M_{12}(t) \quad (10)$$

$$\frac{\mathrm{d}n_{13}(t)}{\mathrm{d}t} = \frac{q_{213}(t)}{\hat{q}_{321}(t) + q_{213}(t) + q_{21}(t)} \cdot u_{21}(t) \cdot M_{21}(t) + q_{13}(t) + q_{131}(t) + q_{132}(t) - \min(M_{13}(t), C_{\mathrm{or},1}(t)) \quad (11)$$

$$\frac{\mathrm{d}n_{21}(t)}{\mathrm{d}t} = q_{21}(t) + q_{213}(t) + \hat{q}_{321}(t) - u_{21}(t) \cdot M_{21}(t) \quad (12)$$

$$\frac{\mathrm{d}n_{22}(t)}{\mathrm{d}t} = \frac{q_{12}(t) + \hat{q}_{312}(t)}{q_{12}(t) + q_{123}(t) + \hat{q}_{312}(t)} \cdot u_{12}(t) \cdot M_{12}(t) + q_{22}(t) + \hat{q}_{132}(t) + \hat{q}_{32}(t) - M_{22}(t) \quad (13)$$

$$\frac{\mathrm{d}n_{23}(t)}{\mathrm{d}t} = \frac{q_{123}(t)}{q_{12}(t) + q_{123}(t) + \hat{q}_{312}(t)} \cdot u_{12}(t) \cdot M_{12}(t) + q_{23}(t) + q_{231}(t) - \min(M_{23}(t), C_{\mathrm{or},2}(t)) \quad (14)$$

$$0 \leq \sum_{j=1}^{3} n_{ij}(t) \leq n_{i,\mathrm{jam}} \qquad i = 1, 2 \quad (15)$$

$$u_{\min} \leq u_{ij}(t) \leq u_{\max} \qquad i = 1, 2;\ j = 3 - i \quad (16)$$

$$u_{\min} \leq u_{\mathrm{or},i}(k) \leq u_{\max} \qquad i = 1, 2; k = 0, 1, \ldots, K-1 \quad (17)$$

$$n_{ij}(t_0) = n_{ij,0} \qquad i = 1, 2;\ j = 1, 2, 3 \quad (18)$$

$$n_{\mathrm{or},i}(t_0) = n_{\mathrm{or},i,0} \qquad i = 1, 2 \quad (19)$$

and (1) – (7)

where t_f [sec] is the final time; $n_{ij,0}$, $i = 1, 2$; $j = 1, 2, 3$, and $n_{\mathrm{or},i,0}$ are the initial accumulations for the MFDs and on-ramps at t_0; $n_{1,\mathrm{jam}}$ and $n_{2,\mathrm{jam}}$ [veh] are the accumulations at the jammed density in regions 1 and 2; $u_{\min}$ and $u_{\max}$ are the lower and upper bounds for perimeter and metering controllers; and $C_{\mathrm{or},i}(t)$ [veh/sec] is the available flow capacity in the on-ramp queue, i.e $C_{\mathrm{or},i}(t) = (n_{\mathrm{or},i,\max} - n_{\mathrm{or},i}(k))/T_k$. Recall that $M_{ij} = (n_{ij}/n_i) \cdot G_i(n_i(t))$, $i = 1, 2; j = 1, 2, 3$. The equations (9)–(14) are the conservation of mass equations for $n_{ij}(t)$, while (15) are the lower and upper bound constraints on accumulation in region i.

3.2 Problem solution – a receding horizon (RH) controller

The optimal control problem is solved following the receding horizon (RH) scheme, where at each time step an optimal open-loop of the problem with finite horizon is optimized, then only the first controller is applied to the plant and the procedure is carried out again. A receding horizon framework has been used for optimization in different traffic control problems, e.g. ramp metering of freeway networks in Bellemans et al. (2006); Papamichail et al. (2010), and mixed urban and freeway networks in van den Berg et al. (2007).

The RH controller obtains the optimal control sequence for the current horizon by solving an optimization problem using the direct sequential method, also referred to as single-shooting or control vector parameterization (CVP) in the literature, e.g. Betts (2010). The direct sequential method transcripts the open-loop optimal control problem into a finite-dimensional nonlinear problem through discretization of the control variables *only* with piecewise constant controls, while the ODEs are embedded in the nonlinear problem, i.e. numerical integration are used between the time steps. The full description of the solution method is not presented here because of the limitation of number of pages, however, the reader can refer to Haddad et al. (2012) for further information.

3.3 Greedy control (GC) and ALINEA

In order to analyze the performance of the *RH* controller, comparison results are done with a state feedback controller that applies a *greedy control* (GC) for the urban network and ALINEA ramp metering strategy for the freeway.

The GC policy for the perimeter controllers is determined by the current accumulations $n_1(t)$ and $n_2(t)$. Let $n_{1,\mathrm{cr}}$ and $n_{2,\mathrm{cr}}$ [veh] be the accumulations that maximize G_1 and G_2, respectively. The GC is designed according to the following policy: if both regions are uncongested, i.e. $n_1(t) \leq n_{1,\mathrm{cr}}$ and $n_2(t) \leq n_{2,\mathrm{cr}}$, then both controllers should maximize the transfer flows, therefore $[u_{12}(t), u_{21}(t)] = [u_{\max}, u_{\max}]$. If one region is congested and the other one is uncongested, then the controllers should minimize the transfer flow to the congested region and maximize the transfer flow to the uncongested region. If both regions are congested, then controllers should minimize the transfer flow to the "more congested" region, and maximize the transfer flow to the "less congested" region, e.g. if $n_1(t)/n_{1,\mathrm{jam}} > n_2(t)/n_{2,\mathrm{jam}}$, then region 1 is more congested than region 2, therefore $[u_{12}(t), u_{21}(t)] = [u_{\max}, u_{\min}]$.

The ALINEA policy for the on-ramp metering controllers is determined by the current accumulation of the cell l connected to the on-ramp. The on-ramp metering controller κ [1/sec] determines the number of vehicles entering the freeway corresponding to the difference between the current accumulation in cell l and the desired accumulation x_{ref},

$$f_{\mathrm{or},l}(k+1) = f_{\mathrm{or},l}(k) + \kappa \cdot (x_l(k) - x_{\mathrm{ref}}) \tag{20}$$

4. RESULTS

In this section, a numerical example with different levels of demand is presented to investigate the characteristics of the proposed *RH* controller. The time varying demand is simulating an one hour of morning peak situation where region 2 as the central business district (CBD) attracts most of the trips. For the presented example, both regions have the same MFD consistent with the MFD observed in Yokohama (Geroliminis and Daganzo (2008)), the lower bound is $u_{\min} = 0.1$, and the upper bound is $u_{\max} = 0.9$. Furthermore, the freeway has four lanes and consists of 17 cells each has length of 0.48 [km] except the first cell which is long enough to accommodate all the vehicles in the entrance queue of the system. The parameters of the triangular fundamental diagram of the freeway cells are: the jam accumulation $\bar{x}_l = 60$ [veh/lane], the mainline capacity $\bar{f}_l = 2000$ [veh/hour/lane], and the free flow speed equals to 88.5 [km/hour]. Other setup of the simulation are: the split ration of off-ramps $\beta = 0.1$ [$-$], the maximum queue size of on-ramps $n_{\mathrm{or},i,\max} = 300$ [veh], and the on-ramps capacity flow $s_{\mathrm{or},i}(k) = 6000$ [veh/hour].

In this numerical example, regions 1 and 2 are initially congested and uncongested, respectively, i.e. the initial accumulations are $n_1(t_0) = 5800$ and $n_2(t_0) = 3500$. The evolution of accumulations $n_{ij}(t)$ over one hour, corresponding to the RH controller with $\theta = 0.7$ are presented in Fig. 3(a), while the evolutions presented in Fig. 3(b) are corresponding to the GC. Note that at the beginning of the control process, both GC and RH controllers decrease the total accumulation in region 1, $n_1(t)$, by $u_{21}(t) = u_{\min}$. Afterwards, the RH controller tries to keep the both accumulations constant by changing u_{21} from $u_{\min}$ to $u_{\max}$ at $t = 240$ to let more vehicles enter to region 1. In contrast, the GC brings the two accumulations equal, i.e. $n_1(650) = n_2(650) = 4200$, and after that instance, both region accumulations increase together while the chattering behavior occurs as a result of switching control between $u_{\min}$ and $u_{\max}$ since the more congested region is constantly altering between region 1 and region 2; note the saw lines of accumulations after $t = 650$. This chattering behavior of GC always happens once both regions reach the critical accumulation and makes both regions to get more congested.

The control sequences $u_{12}(k)$, $u_{21}(k)$, $u_{\mathrm{or},1}(k)$, and $u_{\mathrm{or},2}(k)$ of RH and GC are shown in Fig. 3(c) and Fig. 3(d). The chattering behavior of GC urban perimeter controllers is apparent in Fig. 3(d). The effect of ramp controllers on the condition of freeway is more comprehensible with the help of Fig. 3(e), where the density contour of freeway for RH and GC is illustrated. The main points of interest are cell 3 and cell 11 to the end of freeway specifically during the last 1000 seconds of the simulation. The RH control $u_{\mathrm{or},1}(k)$ by letting more vehicles to enter the freeway than GC tries to avoid spillback from the on-ramp (cell 3) and consequently less vehicle in the region 1 and more vehicle queuing in the freeway behind the cell 3, which seems to be intuitive for

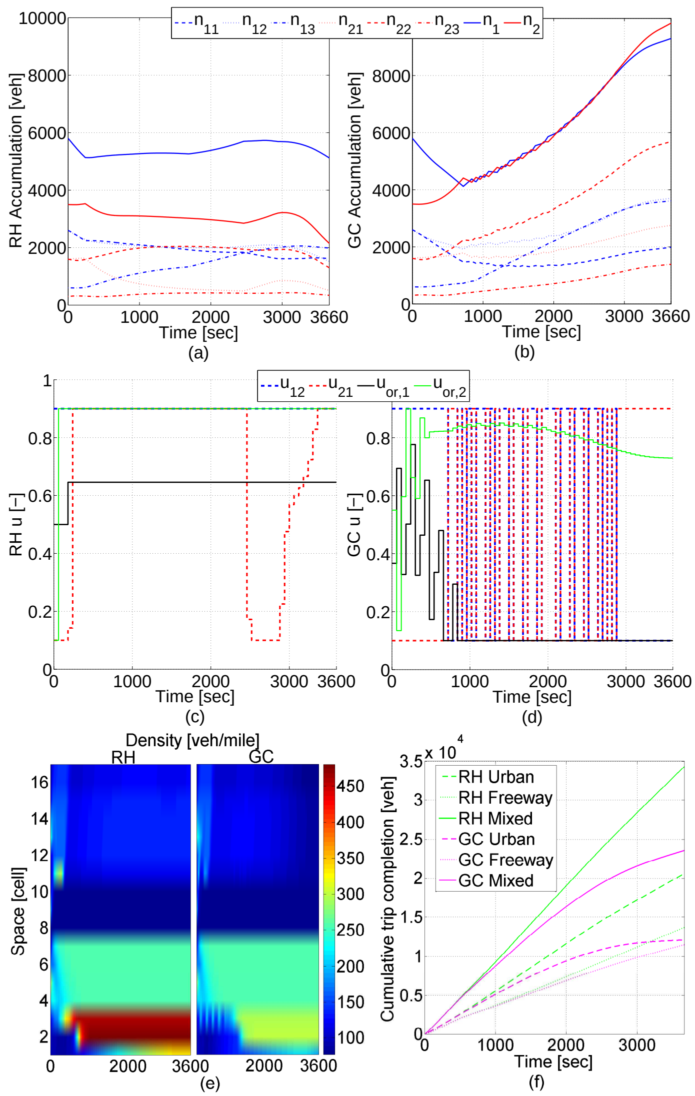

Fig. 3. Numerical example results.

traffic control to keep the vehicles in the freeway instead of urban network during the rush hour. The same behavior is seen for $u_{\mathrm{or},2}(k)$ where the GC tries to keep the cell 11 at capacity which yields to underutilization of the rest of cells to the end of freeway. Note that cells 3 and 11 have on-ramps and cells 7 and 15 have off-ramps and the critical density of cells is 99.4 [veh/km].

The cumulative trip completion corresponding to GC and RH controllers are shown in Fig. 3(f). The key difference is between the urban trip completions which for the GC the almost horizontal ending part of trip completion reveals gridlock of urban network. The difference between freeway trip completions is mostly because of the inherent view of control schemes: RH global view versus local view of ALINEA. Since the second term of objective function is the last cell of freeway outflow, the RH controller tries to make the last cell at capacity, while the goal of ALINEA is to make the cells with on-ramps at capacity (see density of cell 17 specifically during last 1000 seconds of simulation). The effect of demand level on the controllers is also scrutinized; the same initial accumulations with 85% (medium demand) and 70% (low demand) of the original demand. Table 2 summarizes the cumulative trip completion and the difference of the total delays at the end of the simulation (expressed by the area between the RH and GC cumulative trip completion curves [veh · sec]). The consistent results of the example for different levels of demand indicate the superiority of RH controller compared to GC according to the trip completion and total delay. It can be inferred that the differences between the total delays are proportional to the congestion level, i.e. as the demand increases the difference of total delay also increases. In addition, during uncongested regime the performances of GC and RH controller are similar, however, with increase of demand the performance of GC declines, whereas, outcome of RH controller demonstrate that with proper control scheme more traffic can be managed and accommodated in the mixed traffic network.

Table 2. The trip completion and the total delay difference for the numerical example.

demand	RH	GC	RH-GC
	$[\text{veh} \cdot 10^4]$	$[\text{veh} \cdot 10^4]$	$[\text{veh} \cdot \text{sec} \cdot 10^6]$
low	3.29	3.03	5.39
medium	3.47	2.87	8.47
high	3.43	2.36	12.0

5. CONCLUSION

The large-scale control problem of a mixed traffic network consisting of two urban regions with MFD representation and one alternative freeway route modeled with the asymmetric cell transmission model is formulated. For traffic control purposes, two controllers on the perimeter of regions manipulate the urban inter-transfer flow; in addition, two controllers operate on on-ramps to control the traffic flow from urban regions to the freeway. The optimal traffic control problem is solved by a receding horizon control scheme. The proposed controller is compared with an urban perimeter greedy controller plus ALINEA controller for freeway and the results show the advantage of control coordination by the receding horizon controller for all numerical examples with various levels of demand. These results can be beneficial for municipal administrators to develop efficient hierarchical control strategies for mixed traffic networks. Traffic control problem of networks with more complex structure and dynamics is ongoing research.

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