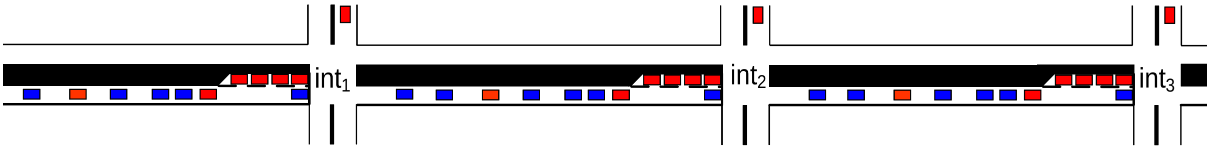
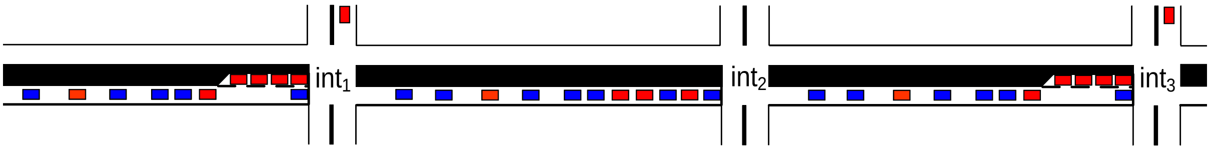


(a) Arterial with permitted left-turn at intersection 2



(b) Arterial with prohibited left-turn at intersection 2

Capacity of arterials with left-turn queue spillbacks

Jack Haddad

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STRC 2012 **May 2012**

Urban Transport Systems Laboratory LUTS

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Capacity of arterials with left-turn queue spillbacks

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Abstract

Traditional capacity calculation of signalized intersections do not take into account the effects of queue spillback that might occur in left-turn bays. As demand of the left-turn increases, and the storage capacity of the bay is limited, the probability of queue spillback occurrence increases. In this paper, we model the effects of queue spillbacks at the left-turn bay on the capacity of the intersection, and as a result we also calculate the capacity drop. Moreover, we extend our model to calculate the capacity of arterials with number of signalized intersections experiencing queue spillbacks. Case study examples examine the effect of intersection characteristics (e.g. left-turn percentage, green durations, and storage capacity) on the arterial capacity drop. Finally, a control strategy that prevents left-turn at intersections with limited storage capacity is proposed to increase the capacity of arterials.

Keywords

signalized capacity, spillback, left-turn bay

1 Introduction

The operation analysis of arterials becomes more challenging as high demand of left turns from the main stream increases. Insufficient queuing space at the left-turn bays of the signalized intersections of the arterial cannot supply the high left-turn demand and queue spillbacks might occur. The queue spillback at the isolated intersection not only affects in a reduction of its capacity, but might also affect the capacity of the arterial. Blocking queue impedes the flow of the main through movement at the arterial, decreases the bandwidths, and increase the delays.

Coordination of intersections has a significant impact on arterials performances as good coordination would enhance their traffic operations. Controlling and managing of intersections at arterials are well understood for undersaturated conditions. The control policy aims to coordinate between signalized intersections as the bandwidth for platoon of vehicles that pass through consecutive intersections is maximized whereby total delay is minimized and capacity is maximized. However, queue spillbacks may occur during congested conditions as demand is higher than capacity and links have limited lengths, therefore, special care should be given to model the effect of queue spillbacks as coordination would be more difficult. Queue spillbacks may also occur during undersaturated conditions if intersections are poorly coordinated.

Queue spillbacks may occur in arterials at: (1) the inter-signal section between intersections, and (2) the short left-turn pocket at downstream intersections. The effects of queue spillbacks at the inter-signal section between intersections were investigated earlier. However, a few research were done to tackle the effects of queue spillback at short left-turn pockets at downstream intersections. The left-turn spillback has a crucial impact on the performance of the arterial especially when the left-turn flow is high and/or variant.

The signal optimization problem has been introduced aiming to maximize the bandwidth or to minimize the intersections delays, e.g. Gartner *et al.* (1991); Little *et al.* (1981). These contributions have not address the issue of having high demand of left turn which may cause for queue spillback, while the model presented recently in Liu and Chang (2011) addresses the queue evolution and capture the queue spillback at lane-groups due to high demand. However, the model describes the interactions between intersections and queue spillbacks in details whereas a large number of parameters and variables are utilized. Other works Prosser and Dunne (1994); Messer (1998); Haddad and Mahalel (2012) address the queue spillback problem at two closely signalized intersections.

The paper is divided into two parts, where in the first part, probability expressions are derived to calculate the capacity in congested conditions for isolated intersections as queue spillback at the short left-turn occurs. Different cases of queue spillback are distinguished whereas blocking queue may occur at the left-turn or the through lane. The differences between capacities with and without consideration of queue spillbacks are demonstrated for different cases. In the second part, the traffic flow dynamics for all intersections in the arterial are integrated together, and case study examples are been performed to demonstrate the effect of different signal-design parameters on the arterial capacity experiencing spillbacks at the left-turn bay. Finally, a control strategy that prevents left-turn at intersections with short storage capacity is demonstrated to show increasing of the arterial capacity.

2 Capacity of isolated signalized intersections with left-turn queue spillbacks

Signal design for isolated signalized intersections is well understandable for undersaturated conditions, i.e. when the demand is less than the capacity, see Webster (1958); Allsop (1972); Improta and Cantarella (1984); Gallivan and Heydecker (1988). For oversaturated conditions vehicle actuated signal control is utilized to minimize the delays by allocating green durations corresponding to the demand. In undersaturated condition the vehicle actuated signal control perform more effectively than oversaturated condition. Increasing the capacity of isolated signalized intersections with separate left turn phases is discussed in Xuan *et al.* (2011). The main idea is to increase the capacity without banning left turns and eliminate wasted green time by introducing a signal light before the intersection. It holds the demand before arriving to the signalized intersection. The problem is solved by linear programming. In Zhang and Tong (2008); Akçelik (1988), probabilistic models that capture the queue spillback at short left-turn bay for one intersection were proposed.

An isolated signalized intersection with left-turn bay is shown in Fig. 1. There is one shared lane that expands to two lanes (left and through movements) as shown in the figure. Let $N_{\max}\,\mathrm{veh}$ be the left-turn lane storage capacity. Queue spillback might occur at the *two-lane section* as queue of vehicles (in the left or through lane) may propagate back and block the flow of vehicles in the *shared-lane section*. Blocking queue prevents movements at the shard-lane section to discharge towards the intersection despite having green period, therefore, part of the green period is not utilized for discharging the demand which results in a reduction of the intersection capacity.

In the following, we deal with two different timing plans for the left-turn approach as shown in Fig. 2, as one has two phases while the other plan has three phases.

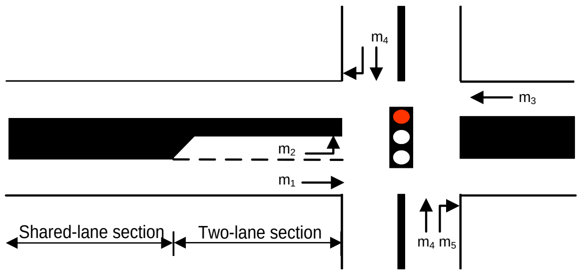


Figure 1: An isolated signalized intersection with left-turn bay

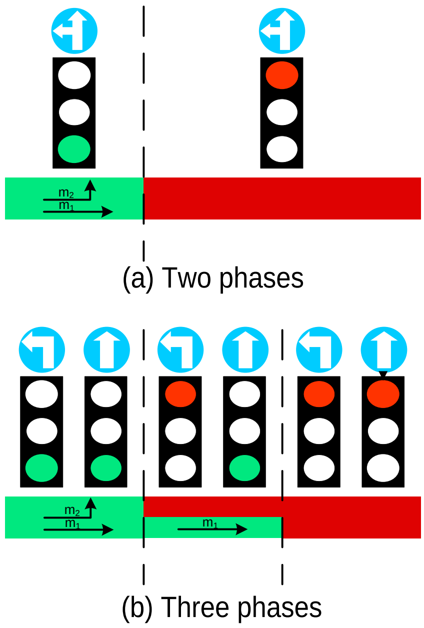
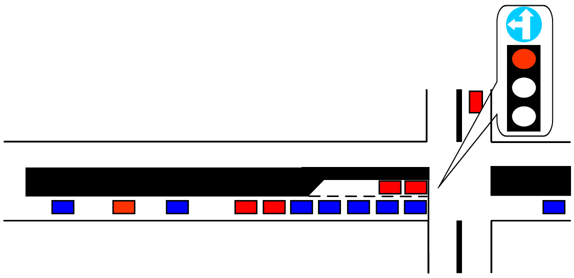


Figure 2: Timing plans for the left-turn approach: (a) two phases, and (b) three phases.

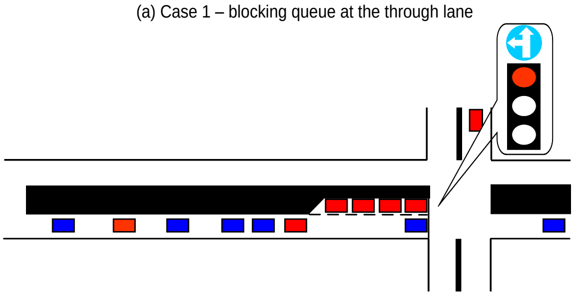
2.1 Two phases for left-turn approach

The left-turn approach has two phases as shown in Fig. 2.(a), one common green, i.e. movements 1 and 2 have green light simultaneously, followed by red green. For the two-phase left-turn approach, the queue spillback at the two-lane section can occur during the red phase in two different cases as shown in Fig. 3: in case 1 a blocking queue propagates at the through lane (movement 1) and prevents the left-turn movement (movement 2) to move forward to the intersection, while in case 2 the queue spillback occurs at the left-turn lane and prevents movement 1 to proceed.

In the following, the expected capacity is estimated by a probabilistic model that takes into account the occurrence probabilities of the two cases 1 and 2. Let us first assume that a blocking queue exists at the through lane. Let $x - 1$ [veh] be the number of vehicles in the two-lane section when a spillback occurs, i.e. sum of total vehicles in both left and through lanes. Note that a blocking queue occurs when there are $N_{\max} + 1$ blocking vehicles from x vehicles.



(a) Case 1 – blocking queue at the through lane



(b) Case 2 – blocking queue at the left-turn lane

Figure 3: An isolated signalized intersection with spillback: (1) case 1 – blocking queue at the through lane, and (2) case 2 – blocking queue at the left-turn lane.

This means that the probability distribution of x is a negative binomial (x the number of trials at which the ($N_{\text{max}}+1$)-th blocking vehicles occurs). For case 1, one can calculate the probability distribution of x as follows

$$f^1(x) = \binom{x-1}{N_{\text{max}}} \cdot (1-p_{\text{D}})^{x-(N_{\text{max}}+1)} \cdot p_{\text{D}}^{N_{\text{max}}+1} \tag{1}$$

where p_{D} $[-]$ is the percentage of vehicles traveling towards the through lane (movement 1) in the platoon arriving from the shared-lane section. When a blocking queue occurs, x has an upper bound $2N_{\text{max}}+1$ (it is not theoretically ∞). Therefore, the expected value of x for case 1, $E^1(x)$ [veh], is calculated as follows:

$$E^1(x) = \sum_{x=N_{\text{max}}+1}^{2N_{\text{max}}+1} x \cdot f^1(x) \tag{2}$$

and the expected value of queuing vehicle in the left-turn lane, $E_T^1(x)$ [veh], is

$$E_T^1(x) = E^1(x) - (N_{\text{max}}+1) \tag{3}$$

Equations (1)–(3) hold for case 2 if the indices 1 and D are switched to 2 and T, respectively.

Under the assumption that the common green period is sufficient to clear the two-lane section spillback, the capacity of the approach for case k, c_k [veh/cycle] $k = 1, 2$, is calculated as follows

$$c_k = E^k(x) - 1 + \left(G_{\mathrm{c}} - \frac{N_{\max}}{s_k}\right) \cdot [p_{\mathrm{D}} \cdot s_{\mathrm{D}} + p_{\mathrm{T}} \cdot s_{\mathrm{T}}] \tag{4}$$

where G_{c} [s] is the common green duration, and s_k [veh/s] is the saturation flow for movement k, $s_1 = s_{\mathrm{D}}$, $s_2 = s_{\mathrm{T}}$. In order to calculate the expected capacity of the approach, one has to calculate the occurrence probabilities of cases 1 and 2, i.e. one has to calculate the probability that a blocking queue occurs at the left-turn lane, Pr_{T} $[-]$, or at the through lane, Pr_{D} $[-]$, where $Pr_{\mathrm{T}} + Pr_{\mathrm{D}} = 1$. The probability of n_T vehicles queue at the left-turn lane, $f(n_T)$ $[-]$, is calculated as follows

$$f(n_T) = \binom{2N_{\max} + 1}{n_T} \cdot p_{\mathrm{T}}^{n_T} \cdot (1 - p_{\mathrm{T}})^{2N_{\max}+1-n_T} \tag{5}$$

A blocking queue at the left-turn lane can occur for $N_{\max} \leq n_T \leq 2N_{\max} + 1$, therefore, the probability of a blocking queue occurs at the left-turn lane is

$$Pr_{\mathrm{T}} = \sum_{n_T = N_{\max}+1}^{2N_{\max}+1} f(n_T) \tag{6}$$

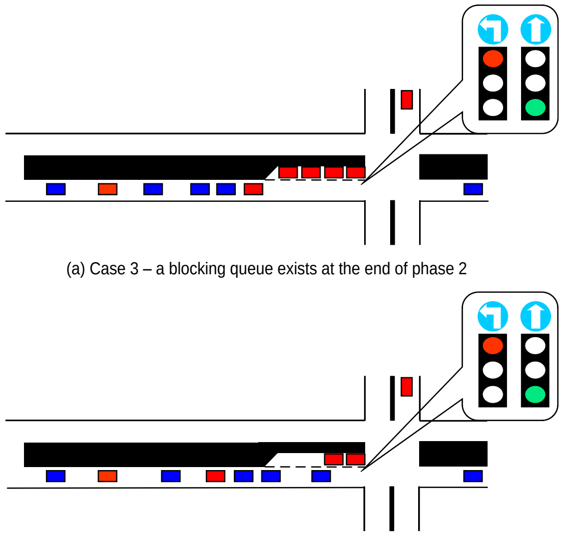
The expected value of the capacity of the approach with left-turn bay, c_{12} [veh/cycle], is calculated as follows

$$c_{12} = c_1 \cdot (1 - Pr_{\mathrm{T}}) + c_2 \cdot Pr_{\mathrm{T}} \tag{7}$$

2.2 Three phases for left-turn approach

In the three phases plan, shown in Fig. 2.(b), there is a through movement phase (phase 2) between the common green (phase 1) and red (phase 3) phases. Calculating the capacity of the left-turn approach having three phases is more complex than the two phases plan. This is because the blocking queue might occur earlier during the through movement phase before the red phase. Moreover, even if the queue spillback does not occur during phase 2, there is a probability that residual vehicles queue at the left-turn lane at the start of phase 3. Hence, the equations for cases 1 and 2 from the previous section must be modified to address the issue of residual vehicles.

During phase 2 the left-turn movement has a red light as vehicles queue behind the stop line of the intersection. Corresponding to the evolution of the queue at the left-turn lane during phase 2, there are two cases as shown in Fig. 4: in case 3 the queue at the left-turn lane propagates to the shared-line and blocks vehicle of the through movement to move towards the intersection, while in case 4 the length of the queue at the end of phase phase 2 is less than the blocking queue, and residual vehicles may queue at the left-turn lane at the start of phase 3.



(a) Case 3 – a blocking queue exists at the end of phase 2

(b) Case 4 – a residual queue exists at the end of phase 2

Figure 4: Queue length at the left-turn lane at the end of phase 2: (a) case 3 a blocking queue exists at the left-turn lane, and (b) case 4 a residual queue.

Let us now consider case 3 where queue spillback occurs during phase 2 as shown in Fig. 4.(a). The maximum number of vehicles in the platoon arriving the two-lane section during phase 2, N_{ph_2} [veh], is calculated as follows:

$$N_{\mathrm{ph}_2} = G_{\mathrm{ph}_2} \cdot s_{\mathrm{D}} \tag{8}$$

where G_{ph_2} [s] is the duration of phase 2. In the following we derive the probability equation for case 3, similarly to case 2. The distribution in case 3 is also a negative binomial distribution, where x the number of trials, and the number of success is $N_{\max} + 1$, as follows:

$$f^3(x) = \binom{x-1}{N_{\max}} \cdot (1 - p_{\mathrm{T}})^{x-(N_{\max}+1)} \cdot p_{\mathrm{T}}^{N_{\max}+1} \tag{9}$$

and the expected value $E^3(x)$,

$$E^3(x) = \sum_{x=N_{\max}}^{N_{\mathrm{ph}_2}} x \cdot f^3(x) \tag{10}$$

where $f^3(N_{\mathrm{ph}_2}) = 1 - \sum_{N_{\max}+1}^{N_{\mathrm{ph}_2}} f^3(x)$. The expected value of vehicles that can discharge before spillback occurs is

$$c_3 = E^3_D = E^3(x) - (N_{\max} + 1) \tag{11}$$

The probability of n_T vehicles queues in the left-turn lane is

$$f(n_T) = \binom{N_{\mathrm{ph}_2}}{n_T} \cdot p_{\mathrm{T}}^{n_T} \cdot (1 - p_{\mathrm{T}})^{N_{\mathrm{ph}_2} - n_T} \tag{12}$$

hence, the probability of a blocking queue occurs at the left-turn lane during phase 2 is equal to

$$Pr_{\mathrm{T}}^{3} = \sum_{n_T = N_{\max}+1}^{N_{\mathrm{ph}_2}} f(n_T) \tag{13}$$

For case 4 where a residual queue exists at the end of phase 2, as shown in as shown in Fig. 4.(b), let $Pr_{\mathrm{v}}(x)$ $[-]$ be the probability of x residual vehicles in the left-turn lane from N_{ph_2} arriving vehicles, and it is calculated as follows

$$Pr_{\mathrm{v}}(x) = \binom{N_{\mathrm{ph}_2}}{x} \cdot p_{\mathrm{T}}^{x} \cdot (1 - p_{\mathrm{T}})^{N_{\mathrm{ph}_2} - x} \tag{14}$$

and

$$E_T^4(x) = \sum_{x=0}^{N_{\max}} Pr_{\mathrm{v}}(x) \cdot x \tag{15}$$

$$c_4 = E_D^4(x) = N_{\mathrm{ph}_2} - E_T^4(x) \tag{16}$$

The capacity during phase 2, c_{ph_2} [veh], i.e. the total number of vehicles discharging and passing the stop line, is

$$c_{\mathrm{ph}_2} = c_3 \cdot Pr_{\mathrm{T}}^{3} + c_4 \cdot (1 - Pr_{\mathrm{T}}^{3}) \tag{17}$$

Let us now consider phase 3. During the red phase (phase 3), the two cases 1 and 2 might happen similar to the two phases plan. If at the end of phase 2, there is a blocking queue from the left-turn lane, then no extra capacity is gained during phase 3. The probability for this to happen is Pr_{T}^{3}, see (13). However, there is a probability of $1 - Pr_{\mathrm{T}}^{3}$ that there exists no blocking at the end of phase 2. Therefore, the later case different number of residual vehicles can be queuing at the left-turn lane.

In the following, we assume that a blocking queue will exist eventually at the end of the common red phase, if it did not happen earlier in phase 2. Sufficient conditions to imply this assumption are that the number of vehicles arriving during common red are sufficient to block downstream, i.e. $N_{\max} > G_{\mathrm{ph}_3} \cdot s_{\mathrm{T}}$ or $N_{\max} > G_{\mathrm{ph}_3} \cdot s_{\mathrm{D}}$.

All vehicles queue during phase 3 (common red) will be discharge in the next cycle during the common green (phase 1). If there is enough green duration during phase 1 to discharge all vehicles queue at the two-lane section, then the expected value of vehicles to discharge is

$$c_{\mathrm{ph}_3} = (1 - Pr_{\mathrm{T}}^3) \cdot \Big[\sum_{i=0}^{N_{\max}} Pr_{\mathrm{v}}(i) \cdot c_{12}(i) \Big] + Pr_{\mathrm{T}}^3 \cdot N_{\max} \tag{18}$$

where i [veh] is the number of residual vehicles in the left-turn lane at the end of phase 2. Otherwise, if there is insufficient green duration to discharge the vehicles, i.e. if $G_{\mathrm{ph}_1} < N_{\max}/s_{\mathrm{T}}$ or $G_{\mathrm{ph}_1} < N_{\max}/s_{\mathrm{D}}$, where G_{ph_1} [veh] is the green duration for phase 1, then

$$c_{\mathrm{ph}_3} = G_{\mathrm{ph}_1} \cdot (s_{\mathrm{T}} + s_{\mathrm{D}}) \tag{19}$$

Note that in this situation we neglect that the rest of the residual vehicles that cannot discharge during the common green in the calculation for phase 2, i.e. we assume no residual vehicles at the start of phase 2.

In order to calculate the expectation of the capacity for cases 1 and 2 during phase 3, $c_{12}(i)$ [veh], one has to calculate the capacity for cases 1 and 2 for i residual vehicles, i.e. $c_1(i)$ and $c_2(i)$, respectively. The calculations are similar to the calculations in Section 2.1, but more complex as they take into account different lengths of residual queue at the left-turn lane on the start of phase 3. Calculation of $c_{12}(i)$ is according to Appendix A.

Recall that during phase 1, some of the green duration is utilized to discharge the blocking queue at the two-lane section. The rest of the common green duration in phase 1 is utilized to discharge vehicles queueing in the shared-lane section. The maximum number of vehicles that can discharge per cycle is

$$c_{\mathrm{ph}_1} = \max(G_{\mathrm{ph}_1} - \frac{N_{\max}}{s_{\mathrm{T}}}, 0) \cdot s_{\mathrm{C}} \tag{20}$$

and the total capacity of the approach, c_a [veh/cycle], is then calculated as follows:

$$c_a = c_{\mathrm{ph}_1} + c_{\mathrm{ph}_2} + c_{\mathrm{ph}_3} \tag{21}$$

and the total capacity for the intersection including all approaches, c_t [veh/cycle], is

$$c_t = c_a + G_{\mathrm{ph}_2} \cdot s_{\mathrm{T}} + 4 \cdot G_{\mathrm{ph}_3} \cdot s_{\mathrm{T}} \tag{22}$$

If the queue spillback at the two-lane section is ignored, then the approach capacity would be bigger. The total capacity of the intersection without considering the queue spillback, $c_{t,\mathrm{ns}}$ [veh/cycle], is calculated as follows

$$c_{t,\mathrm{ns}} = G_{\mathrm{ph}_1} \cdot (s_{\mathrm{T}} + s_{\mathrm{D}}) + 2 \cdot G_{\mathrm{ph}_2} \cdot s_{\mathrm{D}} + 4 \cdot G_{\mathrm{ph}_3} \cdot s_{\mathrm{D}} \tag{23}$$

3 Capacity of arterials experiencing left-turn queue spillbacks

The probabilistic model of isolated signalized intersections presented in the previous section can be utilized to calculate the arterial capacity of several intersections.

For the isolated signalized intersection, it is assume that the approach with left-turn bay has one movement of vehicles as the shared-lane section traveling towards the two-lane section. However, in the arterial as shown in Fig. 5, the shared-lane section is feeded by two movements m_1 and m_5 form the upper stream intersection. Hence, in order to utilize the equations derived in the previous sections, one have to consider the both flows of movements $m_1(j-1)$ and $m_5(j-1)$ (traveling from intersection $j-1$ to j) as a one mixed platoon that has $p_{\text{T,mix}}(j)$ percentage of left-turn at intersection j. First we calculate the number of vehicles discharging from movements m_1 and m_5 at intersection $j-1$ as follows

$$N_1(j-1) = c_{\text{ph}_2}(j-1) + [c_{\text{ph}_1}(j-1) + c_{\text{ph}_3}(j-1)] \cdot p_{\text{D,mix}}(j-1) \tag{24}$$

which is correct under the assumption that a queue spillback occurs at intersection $j-1$. Otherwise,

$$N_1(j-1) = (G_{\text{ph}_1} + G_{\text{ph}_2}) \cdot s_{\text{D}}(j-1) \tag{25}$$

The number of vehicles of movement m_5 discharging from intersection $j-1$ towards intersection j is

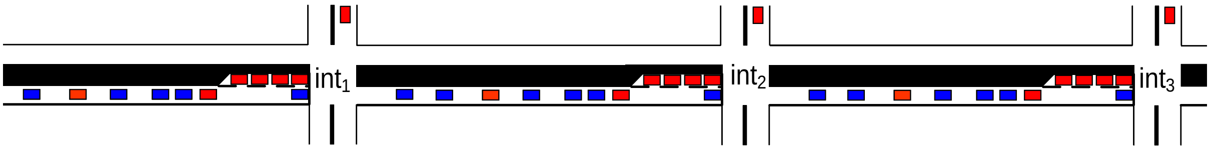
$$N_5(j-1) = G_{\text{ph}_3}(j-1) \cdot s_{\text{D}}(j-1) \tag{26}$$

After calculating $N_1(j-1)$ and $N_5(j-1)$, then one can calculate the percentage of the left-turn vehicles in the mixed platoon as follows

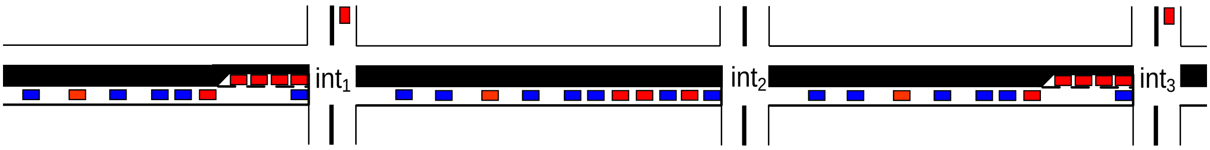
$$p_{\text{T,mix}}(j-1) = \frac{N_1(j-1) \cdot p_{\text{T},1}(j-1) + N_5(j-1) \cdot p_{\text{T},5}(j-1)}{N_1(j-1) + N_5(j-1)} \tag{27}$$

Now we can calculate all $p_{\text{T,mix}}(j)$ for all intersections, and then sum all the intersections capacities to estimate the arterial capacity.

The proposed model is a probabilistic model and can be utilized for design purposes to increase the capacity of arterials. One can prohibit left turn at some intersections with short left-turn bay, that have high probability to queue spillbacks. We can dynamically prevent spillbacks at the left-turn bay, as we assume that we can hold the queue of vehicles in the inter-signal section. We can manage queue. The prohibited left-turn movement in the current intersection can turn left in the subsequent intersections, see e.g. Fig. 5.



(a) Arterial with permitted left-turn at intersection 2



(b) Arterial with prohibited left-turn at intersection 2

Figure 5: Arterial with three intersections where intersection 2 has a: (a) permitted left-turn; (b) prohibited left-turn.

When we prevent the left-turn at an intersection, one has to calculate the left-turn percentage of the mixed platoon similar to (27), however, taking into account that the prevented left-turn flow m_2 will move through at the intersection and will be part of the mixed platoon with m_1 and m_5.

4 Case study examples

In this section, two case study examples are presented. Example 1 deals with an isolated signalized intersection, while example 2 deals with an arterial with three signalized intersections. In example 1 we demonstrate the effect of blocking in decreasing the capacity of the intersection, while in example 2 we demonstrate the effect of prohibiting left-turn movement at the second intersection on the arterial capacity.

In example 1, three phases plan with common green duration $G_{\mathrm{ph}_1} = 30\,\mathrm{s}$, the through movement green $G_{\mathrm{ph}_2} = 30\,\mathrm{s}$, and the red phase duration $G_{\mathrm{ph}_3} = 60\,\mathrm{s}$. The capacity results of the left-turn approach, the intersection, phase 1, and phase 2 are shown in Fig. 6.(a), (b), (c), and (d), respectively. The results are shown for differen sizes of left-turn storage capacity $N_{\max} = 2 - 14\,\mathrm{veh}$.

The effect of the queue spillback at the two-lane section is more apparent in Fig. 7. The green duration for the through movement green varies $G_{\mathrm{ph}_2} = 0,\ 15, 30\,\mathrm{s}$. Note that for $G_{\mathrm{ph}_2} = 0$ the plan has only two phases. The capacity of the left-turn approach and the probability of left-turn blocking queue during phase 2 are shown in Fig. 7.(a) and (b) for $G_{\mathrm{ph}_2} = 0$, in Fig. 7.(c) and (d)

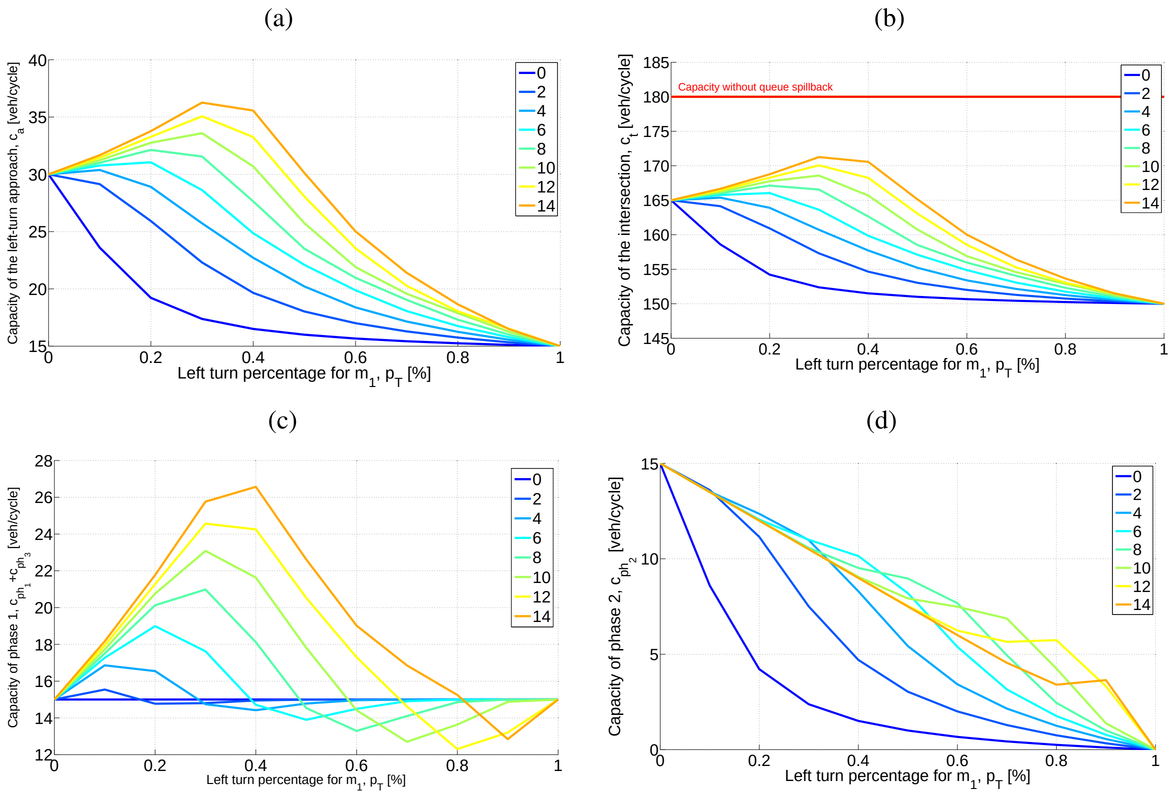


Figure 6: (e) Capacity of the left-turn approach; (b) capacity of the intersection; (c) capacity of phase 1; and, (d) capacity of phase 2, (c_{ph_2})

for $G_{\mathrm{ph}_2} = 15$, and in Fig. 7.(e) and (f) for $G_{\mathrm{ph}_2} = 30$, respectively.

Example 2 presents an arterial with three signalized intersections as shown in Fig. 5. In this example, we examine the effect of preventing the left-turn in intersection 2. The capacities of the arterial are calculated for the three intersections with permitted (dash lines) and prohibited (solid lines) left-turn for different levels of left-turn percentage for m_5 as shown in Fig. 8.(a) for $p_{\mathrm{T},5} = 0.25$, in Fig. 8.(b) for $p_{\mathrm{T},5} = 0.5$, and 8.(c) for $p_{\mathrm{T},5} = 0.75$.

5 Conclusions

A new probabilistic model is presented for signalized intersections with left-turn bays to address the queue spillback effects on the capacity of the intersection. The model is also utilized to evaluate the operational characteristics of arterials with multiple signalized intersections experiencing queue spillbacks. The results presented in this paper demonstrate the effect of spillback on the capacity of the isolated signlaized intersection, and the arterial for different levels of left-turn percentage, different sizes of storage capacity at the bay, and different green durations of phases.

Queue spillback effects the main flow to move smoothly through all intersections in the arterial,

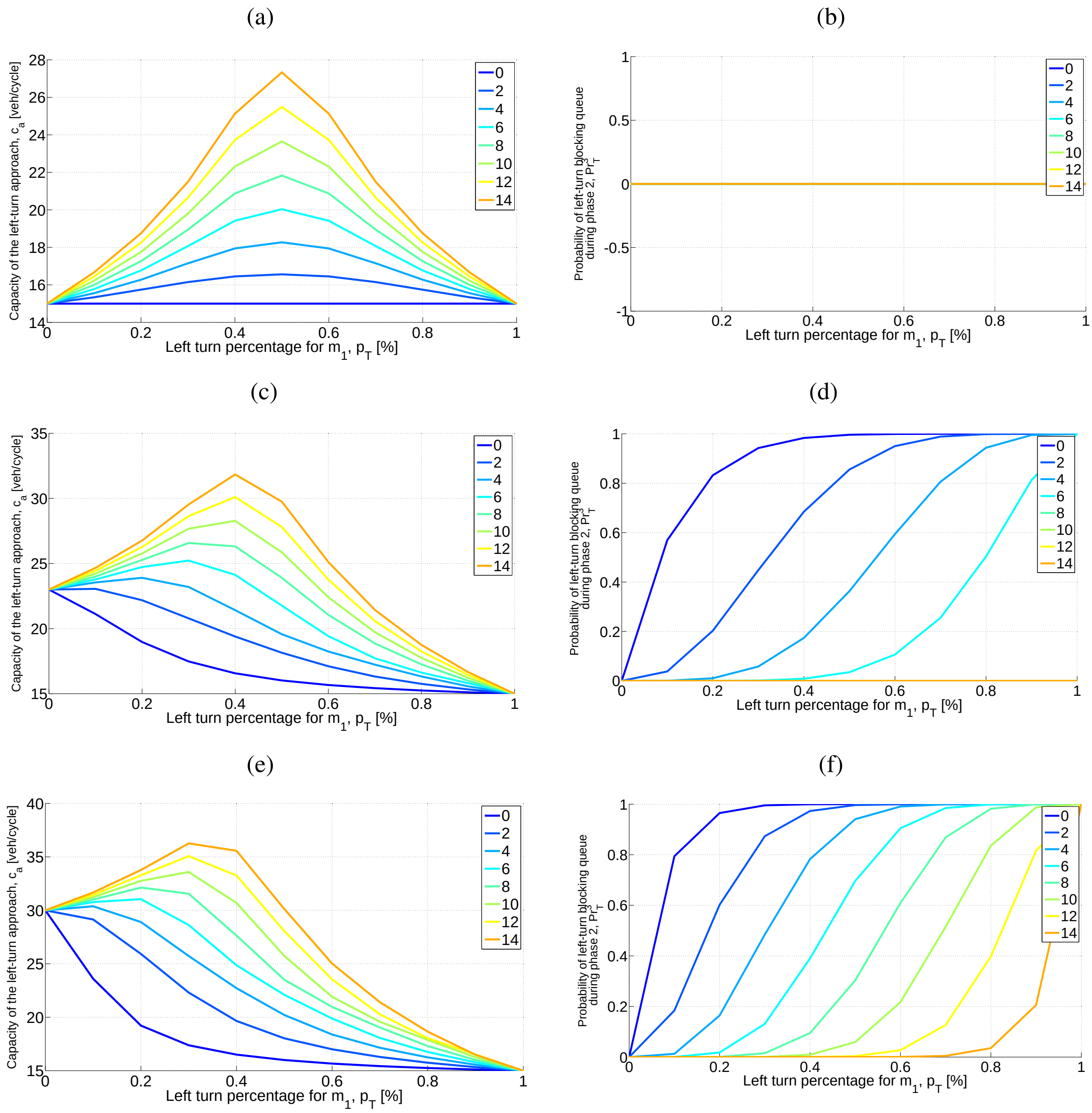
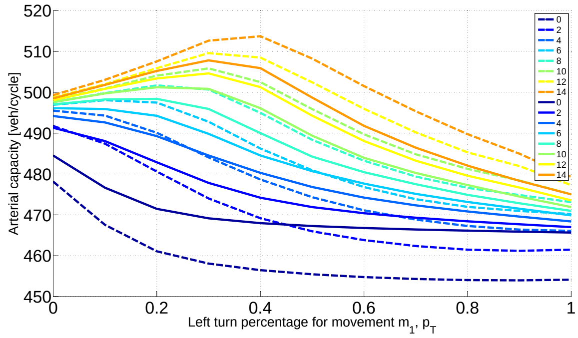


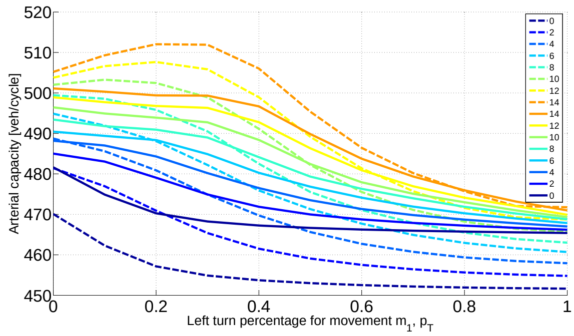
Figure 7: Capacity of the left-turn approach and the probability of left-turn blocking queue during phase 2: (a) and (b) for $G_{\mathrm{ph}_2} = 0$; (c) and (d) for $G_{\mathrm{ph}_2} = 15$; (e) and (f) for $G_{\mathrm{ph}_2} = 30$, respectively.

thus decreasing the bandwidth and the capacity and increasing delays. One can prevent queue spillback by preventing the left-turn at intersections with short left-turn bay and limited capacity storage. In this way, all prevented left-turn vehicles have to change their routes and turn left at the subsequent intersections of the arterial that have higher storage capacity. It was shown that this strategy can increase or decrease the capacity of the arterial depending on the intersection characteristics (e.g. left-turn percentage of the mixed platoon, green durations, and storage capacity) that effect the probability of queue spillback to occur. The proposed model can be applied easily implemented in real-time control of dynamic lane channelization (permit/prohibit left-turn) to increase capacity and decrease delays in arterials.

(a)



(b)



(c)

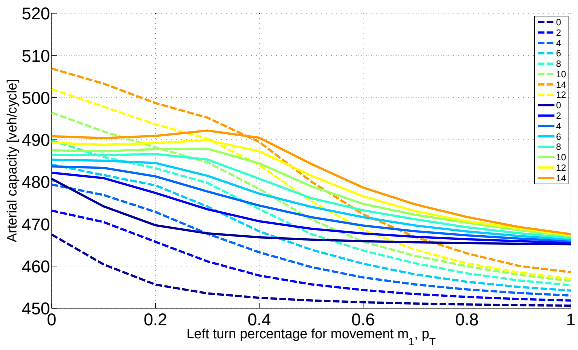


Figure 8: Capacity of arterial experiencing queue spillback with permitted (dash lines) and prohibited (solid lines) left-turn for different levels of left-turn percentage for m_5: (a) $p_{\mathrm{T},5} = 0.25$; (b) $p_{\mathrm{T},5} = 0.5$; (c) $p_{\mathrm{T},5} = 0.75$.

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A Calculation of $c_{12}(i)$

In order to calculate $c_{12}(i)$, one has to calculate $c_1(i)$ and $c_2(i)$.
In case 1, the expected value $E^1(i)$ for a given i is calculated as follows

$$f^1(x) = \binom{x-1}{N_{\max}} \cdot (1-p_{\mathrm{D}})^{x-(N_{\max}+1)} \cdot p_{\mathrm{D}}^{N_{\max}+1} \tag{28}$$

$$E^1(i) = \sum_{x=N_{\max}+1}^{2N_{\max}+1-i} x \cdot f^1(x) \tag{29}$$

therefore, the capacity for case 1 for i is

$$c_1(i) = E^1(i) - 1 + i \tag{30}$$

if $N_{\max} = 0$, then $c_1(i) = \max(E^1(i) - 1, 0)$.

In case 2, the expected value $E^2(i)$ for a given i is calculated as follows

$$f^2(i,x) = \binom{x-1}{N_{\max}-i} \cdot (1-p_{\mathrm{T}})^{x-(N_{\max}+1-i)} \cdot p_{\mathrm{T}}^{N_{\max}+1-i} \tag{31}$$

$$E^2(i) = \sum_{x=N_{\max}+1-i}^{2N_{\max}+1-i} x \cdot f^2(i,x) \tag{32}$$

where $f^2(i, 2N_{\max}+1-i) = 1 - \sum_{x=N_{\max}+1-i}^{2N_{\max}-i} f_2(i,x)$, and

$$c_2(i) = E^2(i) - 1 + i \tag{33}$$

if $N_{\max} = 0$, then $c_2(i) = \max(E^2(i) - 1, 0)$.

The probability of a blocking queue exists at the through lane, Pr_{D}^1(i), is calculated as follows,

$$f_D(i, n_d) = \binom{2N_{\max}+1-i}{n_D} \cdot p_{\mathrm{D}} \cdot (1-p_{\mathrm{D}})^{2N_{\max}+1-i-n_D} \tag{34}$$

then,

$$Pr_{\mathrm{D}}^1(i) = \sum_{n_D=N_{\max}+1}^{2N_{\max}+1-i} f_D(i, n_D) \tag{35}$$

In a similar way, we calculate the probability of a blocking queue exists at the left-turn Pr_{T}^2(i),

$$f(i, n_T) = \binom{2N_{\max}+1-i}{n_T} \cdot p_{\mathrm{T}}^{n_T} \cdot (1-p_{\mathrm{T}})^{2N_{\max}+1-i-n_T} \tag{36}$$

$$Pr_{\mathrm{T}}^{2}(i) = \sum_{n_T=N_{\max}+1-i}^{2N_{\max}+1-i} f_T(i, n_T) \tag{37}$$

The expected value of the capacity for cases 1 and 2 during phase 3 period for a given i is

$$c_{12}(i) = c_1(i) \cdot Pr_{\mathrm{D}}^{1}(i) + c_2(i) \cdot Pr_{\mathrm{T}}^{2}(i) \tag{38}$$