Multi-Objective Airport Gate Assignment Problem

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Abstract

In this paper, we consider the gate assignment for a large airline at its hub airport. It is considered to be a highly complex problem with the possibility of application in both planning as well as operations mode. There are various considerations that are involved while assigning gates to incoming and outgoing turns at an airport. Different gates have restrictions, such as adjacency, LIFO and push time, which is known in advance from the structure of the airport. When optimizing the assignment costs, we consider different and often conflicting objectives such as maximization of gate rest time between two turns, minimization of the cost of towing an aircraft with a long turn and minimization of overall costs that includes penalization for not assigning preferred gates to certain turns.

One of the major contributions of this paper is gate assignment in the planning mode to assign airport gates dynamically to scheduled flights based on daily origin and destination passenger flow data ensuring that the number of passenger misconnects at the hub airport is minimized.

We formulate these problems as mixed 0-1 integer program with a linear objective function and constraints. Due to the complexity in the problem size and formulation, we have resorted to relaxation for certain instances when a reasonable solution is not obtained within the time limit. In order to compare the performance of standard MIP, a set partitioning formulation is proposed. The advantage of attempting this alternate formulation is the capability to compare the performance of a normal MIP solution with a column generation algorithm. Implementation is done using OPL and computational results for actual data sets are presented.

Keywords
Airport Gate Assignment – Scheduling – Mathematical Modelling

1. Introduction

The airline industry has long been a fertile area for applying optimization techniques. In this paper, we consider an optimization problem that allows an airline company to dynamically assign existing airport gates to its scheduled flights based on passengers' daily origin and destination (O&D) data. This paper describes the gate assignment problem as experienced at congested hub airports and large airline companies. In one data set, we attempt to assign about 75 gates to over 1200 flights at a major international hub.

The period of time that an aircraft spends on the ground between arriving and departing flights is called a turn. For every turn, the aircraft is assigned to a gate, and the same gate is utilized by many aircrafts over the course of a day. Airport operations team develop gate assignment plans using an optimization model that assigns gates to every turn, while balancing operational constraints given the fleet and turn information through a station. Each hub airport must have a gate plan based on its geography.

In this paper, we consider the gate assignment for planning mode where cost minimization and revenue maximization are major optimization criteria as opposed to feasibility of solutions. In the planning mode, flight schedule and gate plan are used to arrive at a gate assignment schedule while ensuring that the business constraints are satisfied and objective gets an optimal value. We develop a basic 0-1 integer program mathematical model formulation with linear objective function and constraints that would assign one gate to every flight and ensure that all business constraints are satisfied. Additional objectives in planning and operations mode, which are described as the following, would be added to the basic model as extensions:

1.1 Maximize revenue for tight connections

Connecting passengers at a hub airport generally arrive at one gate and depart from another gate, with a certain planned time for traveling between the gates (or through customs). While passengers are not allowed to book an itinerary with a connection time less than a particular threshold (minimum connect time), in practice many connections are very close to the minimum, such that a small delay can create misconnect liability.

When building a 0-1 integer program formulation, one of the key issues is the choice of decision variable. We consider the gating plan of an incoming flight connection as well as an outgoing flight connection rolled into one variable. Thus, for a flight schedule with 800 flights and 100 gates, the worst case scenario could result in 6.4bn 0-1 variables. Fortunately, every flight does not always present a connection opportunity to passenger with every other flight. Incidentally, a flight can potentially connect to barely 20 other flights and, in many cases, the connection time is often more than the longest walking time between two gates at the airport.

1.2 Operational robustness

Gate rest is a concept that is utilized to improve gate plan robustness. Gate rest is the time when the gate is unoccupied by aircraft - between the departure of one aircraft and the arrival of the next one. The gate plan should consider different minimum gate rest characteristics for Turbo-Props, Regional Jets, Mainline, and International business sectors. It would also dynamically consider gate rest, given expected inbound arrival delays, or other operational characteristics. It should be capable of handling, without excessive disruption, the propagation involved in a typical out of service (OTS) aircraft problem. By incorporating these criteria in the Gate Assignment Problem (GAP), a robust gate plan can be handed off to airport personnel to better manage the gate plan on day of operation.

This paper describes the problem formulation, relaxations, if required, and the model inputs in the next section. Section 3 describes the implementation methodology and some of the key results. This section also discusses the benefits realized by the implementation for the different objectives. Section 4 concludes the paper and gives some direction for future work.

2. Problem Data and Terminologies

Gate Assignment Problem is a NP-Hard problem as it belongs to the class of generalized assignment problems. Before we describe the mathematical model, we would mention the different data sources would be used in the planning and operations mode.

Input data to the model in the planning mode would include the following:

- Turns data (comprising of turn id, incoming flight id, outgoing flight id, departure and arrival times of both incoming and outgoing flights, origin, via and destination stations)
- Flights data (comprising of flight ids, flight numbers, departure and arrival times, origin and destination stations, origin and destination stations, reference day)
- International Routes data (comprising of international stations data)
- Itinerary (Connection) data (comprising of itinerary id, incoming flight id, outgoing flight id, min and max connection times, number of pax, revenue per pax)
- Station Gate data (comprising of gate information, list of equipments allowed for a gate and other miscellaneous restrictions such as adjacency, LIFO or push back)
- Walking Time data (comprising of gate to gate walking time data, equipment-wise deplane data, boarding time data, load-bridge loading time)

Input data to the model in the operations mode would include the following:

- Turns data is the output of the planning model. This file contains all gates to be gated as well as its assigned gates. Only the turns present in this file are considered for re-gating.
- Long turns data used for reporting purposes of a towed long turn

- Station Gate data (comprising of gate information, list of equipments allowed for a gate and other miscellaneous restrictions such as adjacency, LIFO or push back)

Before the more complex objectives as described in section 1 are discussed, we will introduce the basic gate assignment formulation that is solved as a feasibility problem while ensuring that the business constraints are satisfied. Following are the business constraints that need to be considered while formulating the mathematical model:

- **Gate Rest**

  Gate rest is defined as the time duration for which the gate is kept idle between a departure and the next arrival. The purpose of gate rest is to ensure that the gate plan remains fairly robust in the event of minor delays in the flight schedule. A 10 min gate rest ensures that two successive flights are assigned the same gate if and only if the arrival time of the later flight is at least 10 min after the departure time of the former.

- **Adjacency Constraints**

  Adjacency constraint is described as a situation when a gate A is occupied by aircraft type 1, then the adjacent gate B doesn't allow aircraft type 2. Normally, when gate C2 is occupied by any narrow body fleet type then gate C1 is blocked for medium and wide body fleet type.

- **LIFO Constraints**
  LIFO constraints are last in first out; these are applied in a situation where two gates are in front of each other. If the second gate is occupied by an aircraft then the aircraft in first gate cannot depart or be used.

- **Pushback Constraints**
  Push back restrictions are defined in the following manner. If two gates occupy the same push back path, then if one of the aircraft from gate 1 push backs then there as to be minimum push back separation time between aircraft departing from gate 2 and gate 1.

- **Towing**
  Towing means that an aircraft is towed away after it arrives at a gate for passengers to deplane. It will then be towed back to a gate for departure. The departure gate may or may not be the same as the arrival gate. The purpose of towing is to free up a gate for other turns' use, so it is only worthwhile to tow turns with a long duration, i.e., a long turn. Currently, long turn is defined to be a turn with the turn time greater than or equal to 120 minutes. A towing operation is shown in the figure below

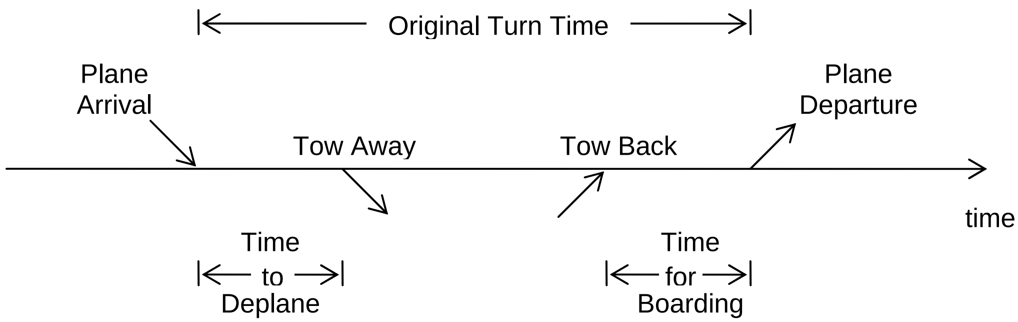

Fig 1: Towing Representation

Conceptually, the time period between arrival and towing away can be modeled as a turn. The duration for which the aircraft occupies the gate can be thought of the turn time. The tow which takes the aircraft away can be thought of as a departure flight. The time at which the aircraft is towed away can be thought of as the departure time. Similarly, the time period between towing back and departure can also be modeled as a turn.

We now present a 0-1 integer program that would help us produce a feasible gate plan in the light of all the above business constraints. We first describe the parameters, decision variable and their notations and then present the model.

3. Mathematical Model

Parameters

α: minimum gate rest

β: minimum push-back separation

ξ: maximum number of turns allowed to be unassigned

The following are the data sets for the schedule and the airport:

T: set of turns to be gated

T_L: set of long turns that may be towed ($T_L \subset T$)

K: set of gates

J: set of adjacent gate pairs that have the adjacent gate restriction

J^{PB}: set of adjacent gate pairs with pushback restrictions

LF: set of front and rear gate pairs

E_k: set of equipment types that gate k can handle, $k \in K$

The following are the parameters corresponding to schedule and gates

a_i: scheduled arrival time of turn $i \in T$

b_i: scheduled departure time of turn $i \in T$

$p_i \in T_L$: new turn from long turn $i \in T_L$

e_i: equipment type used by turn $i \in T$

E_k: set of equipment types that gate k can handle, $k \in K$

k, l: two gates restricted in the adjacent pair $(k,l) \in J$

E_k^1, E_l^1: sets of equipment types such that when an aircraft of a type in E_k^1 is occupying k, no aircraft of any type in E_l^1 may use l; and vice versa.

k^{PB}, l^{PB}: two adjacent gates pair with pushback restriction $(k^{PB}, l^{PB}) \in J^{PB}$

k_{LiFo}^F, l_{LiFo}^R: a pair of front and rear gates in $(k_{LiFo}^F, l_{LiFo}^R) \in LF$, where k_{LiFo}^F is the front gate and l_{LiFo}^R is the rear gate

$E_{k^{LiFo}}^F, E_{l^{LiFo}}^R$: sets of equipment types such that when an aircraft of a type in $E_{k^{LiFo}}^R$ is occupying a rear gate k_{LiFo}^R, no aircraft of any type in $E_{l^{LiFo}}^F$ may arrive or depart at k_{LiFo}^F, $(k_{LiFo}^F, l_{LiFo}^R) \in LF$

Decision Variables

$x_{ik} \in \{0,1\}$: 1 if turn *i* is assigned to gate *k*; 0 otherwise

$y_i \in \{0,1\}$: 1 if turn *i* is not assigned to any gate; 0 otherwise

$w_t \in \{0,1\}$: 1 if long turn *t* is towed; 0 otherwise

Objective Function

Maximize $\sum_{i \in T} \sum_{k \in K} C_{ik} x_{ik} - C_1 \sum_{t \in L} w_t - C_2 \sum_{i \in T} y_i$

C_{ik} represents how favorable or unfavorable it is to assign turn *i* to gate *k*. This coefficient is affected by a number of business and operation preferences:

1. Some pre-defined sets of conveniently located gates are preferred for top Business market flights and Premium Services flights
2. Some schengen gates are capable of accommodating non-schengen arrivals, but they are less preferred than gates at non-schengen terminals
3. Some gates are less preferred for some fleet types because of gate features

C_1 represents the unit cost of towing a turn

C_2 represents the unit cost of not assigning a gate to a turn

Constraints

subject to:

$$\sum_{k \in K} x_{ik} + y_i = 1, i \in T \tag{1}$$

$$\sum_{k \in K: e_i \in E_k} x_{ik} + y_i = 1, \qquad i \in T \tag{2}$$

$$\sum_{i \in T} y_i \le \xi \tag{3}$$

$$y_{ik} + y_{jk} \le 1, \qquad i,j \in T; k \in K : a_i < b_j + \alpha, a_j < b_i + \alpha, i \neq j \tag{4}$$

$$y_{ik} + y_{jl} \le 1, \qquad i,j \in T; k,l \in K; (k,l) \in J : a_i < b_j, a_j < b_i, \; i \neq j, e_i \in E_k^1, e_j \in E_l^1 \tag{5}$$

$$y_{ik} + y_{jl} \le 1, \qquad i,j \in T; k,l \in K; \left(k_{LiFo}^F, l_{LiFo}^R\right) \in LF : a_j \le a_i \le b_j, i \neq j, \; e_i \in E_{k^{LiFo}}^F, e_j \in E_{l^{LiFo}}^R \tag{6}$$

$$y_{ik} + y_{jl} \le 1, \qquad i,j \in T; k,l \in K; \left(k_{LiFo}^F, l_{LiFo}^R\right) \in LF : a_j \le b_i \le b_j, i \neq j, \; e_i \in E_{k^{LiFo}}^F, e_j \in E_{l^{LiFo}}^R \tag{7}$$

$$y_{ik} + y_{jl} \le 1, \qquad i,j \in T; k,l \in K; \left(k^{PB}, l^{PB}\right) \in J^{PB} : b_i - \beta < b_j < b_i + \beta, \; i \neq j, e_i \in E_k, e_j \in E_l \tag{8}$$

$$w_t \le \tau, \qquad t \in T_L \tag{9}$$

$$y_{i_1 k} - y_{i_2 k} \le w_t, i_1, i_2 \in T, t \in T_L, k \in K : i_1 \neq i_2, p_{i_1} = p_{i_2} = t \tag{10}$$

$$y_{i_1 k} + y_{i_3 k} - 1 \le w_t, \qquad i_1, i_2, i_3 \in T; t \in T_L; k \in K : a_{i_1} < a_{i_3}, b_{i_3} < a_{i_2}, \; p_{i_1} = l, p_{i_2} = l \tag{11}$$

$$y_{i_1 k_1} + y_{i_3 k_2} - 1 \le w_t, \qquad i_1, i_2, i_3 \in T; t \in T_L; k_1, k_2 \in K; j \in J : a_{i_1} < a_{i_3}, b_{i_3} < b_{i_2}, \; p_{i_1} = l, p_{i_2} = l, k_1 = q_j^1, k_2 = q_j^2, e_{i_1} \in E_j^1, e_{i_2} \in E_j^2 \tag{12}$$

(1) ensures that turn *i* is assigned to at most one gate.
(2) ensures that turn *i* is assigned to a gate only if its equipment type is among the types which the assigned gate can accommodate.
(3) restricts the number of ungated turns to less than or equal to the allowed number ξ
(4) ensures that at any given time, at most one turn is assigned to one gate.
(5) ensures that adjacency constraints are observed.
(6) – (7) enforces LIFO restrictions. Arrivals and departures at the front gate are prohibited by (6) and (7), respectively, when the rear gate is occupied.
(8) ensures that pushback restrictions are observed.
(9) ensures that no turn is towed if towing is not allowed.
(10) – (12) ensures that if a long turn *t* is towed, the w_t variable is set to be 1:

- (10) If at any gate, only one of the two half-turns (i_1, i_2) of a long turn *t* is assigned but not both, *t* is towed.
- (11) If a) i_1 is the first half-turn of a long turn *t*, and
  b) the durations of *t* and i_3 overlap with each other,

then i_1 and i_3 may be assigned to the same gate only if *t* is towed.

- (12) If a) i_1 is the first half-turn of a long turn *t*,
  b) the durations of *t* and i_3 overlap with each other, and
  c) the equipment types of i_1 and i_3 would violate the adjacency conflict at k_1 and k_2,

  then i_1 and i_3 may be assigned to k_1 and k_2, respectively, only if *t* is towed.

3.1 Model Extension to Maximize Passenger Connection Revenue:

In this model, we aim to maximize the revenue by minimizing the passenger misconnects due to connection flights being assigned to distant gates.

Though there are stipulated minimum and maximum connection times, it must be noted that flights gated at distant gates could possibly result in misconnects if the connection time is fairly tight. For a passenger who transfers to a connection flight at the airport, the connection time is readily defined as the walking time required from the arrival gate of her incoming flight to the departure gate of her outgoing flight. For the planning model, the arrival time of the incoming flight and the departure time of the outgoing flight are fixed due to the published flight schedule. Each flight must be assigned to exactly one gate, and there should be sufficient time for passengers boarding at the gate.

We extend the basic model given above to incorporate this objective in the mathematical formulation

Sets:
FLIGHTS: Set of all flights
TURNS: Set of all turns involving flights f_1 and f_2, i.e., (f_1,f_2)
GATES: Set of all gates
CNX: Set of all revenue connections involving turns i and j, i.e., (i,j)
$\mathrm{GATES}_i^{'}$: Gates allowed for the turn i
AGP: Sets of adjacent gate push times
AGR: Set of adjacent gate restrictions

Parameters:
$REVENUE_{ij}$: Revenue generated by connecting turn i to turn j. Note that $(i,j) \in$ CNX. This can be calculated dynamically by considering transit time between gates k and l, number of connecting passengers, revenue generated and max connection time allowed.
$Arrival(i)$: Arrival time of turn i

$Departure(i)$: Departure time of turn i

$Walk(k,l)$: Wholesome walking time including boarding, de-boarding and other components of time

Variables:

z_{ijkl} : 1 if turn i is assigned to gate k and turn j is assigned to gate l and *(i,j)* ϵ CNX, 0 otherwise

y_i : 1 if gate is not assigned to turn i, 0 otherwise.

x_{ik} : 1 if turn i is assigned gate k, 0 otherwise.

Mathematical Model:

Maximize

$$\sum_{(i,j)\in CNX} \sum_{k\in GATES} \sum_{l\in GATES} REVENUE_{ij} z_{ijkl} \tag{13}$$

Subject to

$$\sum_k z_{ijkl} \le \sum_k x_{ik} \quad \forall \quad (i,j)\in \mathrm{C}NX \quad \wedge \quad i,j\in TURNS \quad \wedge \quad l\in \mathrm{GATES} \tag{14}$$

$$\sum_l z_{ijkl} \le \sum_l x_{jl} \quad \forall \quad (i,j)\in \mathrm{C}NX \quad \wedge \quad i,j\in \mathrm{TURNS} \quad \wedge \quad k,\in \mathrm{GATES} \tag{15}$$

$$x_{ik} + x_{jl} - 1 \le z_{ijkl} \quad \forall \quad (i,j)\in \mathrm{C}NX \quad \wedge \quad i,j\in \mathrm{TURNS} \quad \wedge \quad k,l\in \mathrm{GATES} \\ \wedge\ Arrival(i) + Walk(k,l) \le Departure(j) \tag{16}$$

$$z_{ijkl} = 0 \quad \forall \quad (i,j)\in \mathrm{C}NX \quad \wedge \quad i,j\in \mathrm{TURNS} \quad \wedge \quad k,l\in \mathrm{GATES} \\ \wedge\ Arrival(i) + Walk(k,l) > Departure(j) \tag{17}$$

$$z_{ijkl} \in \{0,1\} \quad \forall \quad (i,j)\in \mathrm{C}NX \quad \wedge \quad i,j\in \mathrm{TURNS} \quad \wedge \quad k,l\in \mathrm{GATES} \tag{18}$$

(13) is the objective function addition to the existing model to come up with the new model. We maximize the overall revenue by realizing the connection revenue between flights which are components of turns i and j. Revenue computation would be done carefully by looking at the possibility of connection from i to j as well as j to i. In addition, penalty for any unassigned flight in kept at minimum, preferably zero, for the planning problem.

(14) and (15) are upper bounds that ensure x_{ijkl} would not take a value of 1 unless both turns i and j are assigned to gates k and l respectively. Note that this is a necessary condition for this variable to take a value 1, but by no means sufficient. (16) is a lower bound for this variable. This variable x_{ijkl} is created for only “select” valid connections between turns i and j. By “select” connections, we refer to those connections where the connection time is greater the minimum time for a passenger to de-board, walk and board another connection while less than the maximum time for a passenger to de-board, walk and board another connection for a given hub airport. For instance, the maximum time for a passenger to de-plane, walk and board another flight at this hub is 41 min for schengen connections irrespective of the equipment type. It is not difficult to prove that we do not sacrifice optimality by such an assumption because any connection with connection time less than minimum walking time

would never materialize even if the corresponding equipments are gated at closest gates. Similarly, there would be no adverse impact on the revenue by assigning connecting turns to farthest gates (or any other combination of gates) if the total connection time is greater than the maximum walking time between two gates at the airport.

(18) would ensure that the decision variable z_{ijkl} takes a value 1 if and only if walking times between flights involved in turns i and j is less than the difference between departure and arrival flights in turns j and i respectively.

Note that these features and constraints are only addition to the existing model. There is no change in the basic framework of the existing model and the new model only introduces several new variables and constraints that would help us solve the objective of maximizing connection revenue.

3.2 Model Extension to Optimize Gating Robustness:

We use the same existing model to optimize gating robustness with a minor change. We create a delay variable, *delay,* for a particular turn to include the extent of flight delay in both arrival and departure. *delay* for a turn *i* is computed as:

$delay_i = \text{Max}(Arrival\ Delay_i, Departure\ Delay_i)$

We use the following addition to the existing model:

$$Objective(Maximize):$$

$$-GR\_PENALTY \sum_{i \in TURNS} gr\_violation\_ttl_i$$

$$Constraint:$$

$$(y_{ik} + y_{jk} - 1).(Departure_i + Gate\_\text{Re}st + delay_i - Arrival_j) \leq gr\_violation\_ttl_i \qquad \forall i, j \in Turns$$

4. Implementation and Results

The models described in Section 2 are implemented in Optimization Programming Language (OPL) and run on a SUSE-Linux server with 4 GB RAM. The sample data set used by us had 605 turns, 25414 flight connection opportunities accounting for revenue of over €30mn and 75 gates available at the hub airport under consideration.

We also implemented the following solution considerations to the planning model objectives to make the model simpler and eliminate large number of unnecessary variables.

- Only those connections which are potentially feasible are considered

- All connections where the connection time is lesser than walk time between nearest gates + boarding time + deplane time + load bridge time are ignored and assumed to be lost revenue in any case.
- All connections whose connection time is greater than walk time between farthest gates + boarding time + deplane time + load bridge time are ignored and assumed to be realized revenue in any case.
- If a connection is realized, total revenue from that connection is assumed to be realized and vice-versa.
- Minimum gate rest for mainline and shuttle are provided for all cases.
- The other two levels (Gate rest for mainline, shuttle and delays) are given same incentive.

The implementation of the above-mentioned solution considerations reduced the number of critical (or at risk) connections substantially and the corresponding revenue to $8.5mn. For benchmarking the various planning objectives, we consider the following scenarios:

- Basic gate assignment model, solved with feasibility objective
- Maximization of passenger connection revenue while assigning gates
- Maximization of gate rest (Robustness of the gate plan solution), minimization of zone usage and maximization of connection revenue while assigning gates

The primary criteria in all these objectives are to ensure that every flight is gated. This is done by heavily penalizing ($1mn) any un-gated turn. In the event of multiple objectives, violation of every objective is penalized depending on its importance to the user. Figures 2 and 3 present a comparative picture of the different objectives for the given dataset. As expected, run times increase with the complexity of objectives, the problem size, the number of shifts under consideration and the threshold for minimum gate rest for different types of aircrafts. It is preferred to keep the run times within 30-40 min.

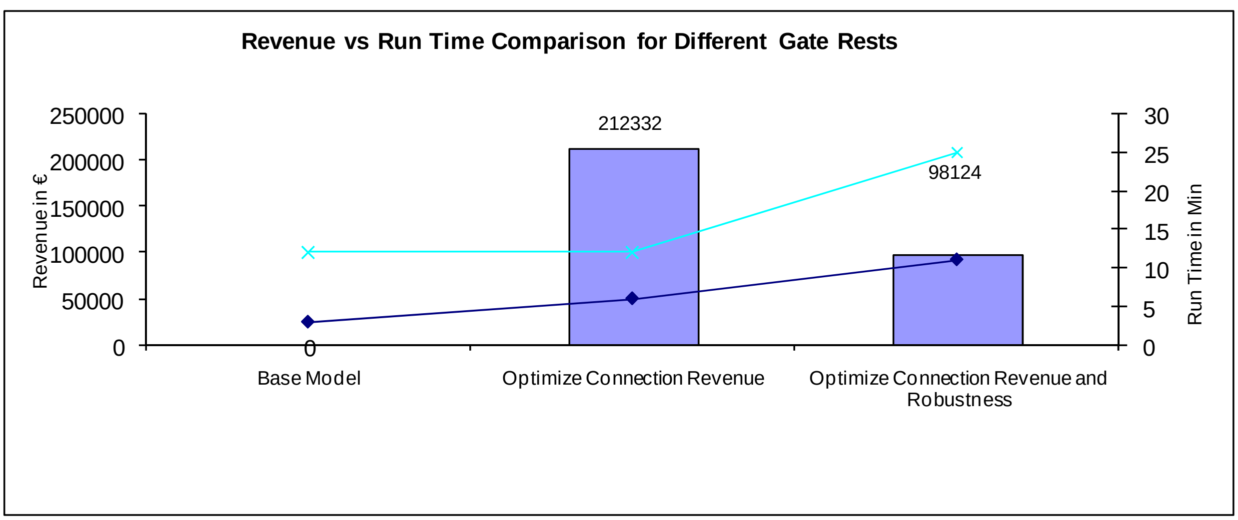

Fig 2: x min of gate rest for Turbo-propellers and Regional Jets, y min of gate rest for narrow and wide body aircrafts

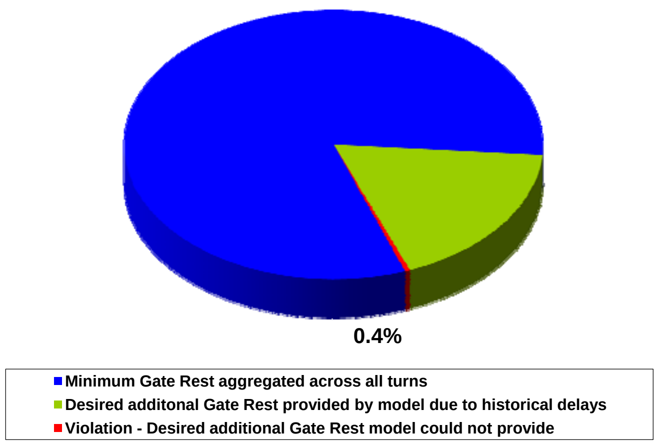

Fig 4: Desired gate rest is obtained based on historical pattern of delays; violation is only 0.4% when evaluated separately as well as with other objectives

5. Concluding Remarks

We have presented the airline gating problem as observed by the industry. The problem considered in this paper is actually implemented for a very large airline. One of the major contributions of this paper is the introduction of different types of planning and operations objectives for this problem that are observed in the real-life to the reader.

Mathematical modeling and computational experience for these different objectives is another original contribution of this paper to the airline and gate assignment research. This paper has also indicated to certain objectives where it is difficult to obtain a solution without relaxation.

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