Random Sampling of Alternatives for Route Choice Modeling

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Conference paper STRC 2007

STRC
7th Swiss Transport Research Conference
Monte Verità / Ascona, September 12. – 14. 2007

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September 2007

Abstract

In this paper we present a new point of view on choice set generation for route choice models. When modeling route choice behavior using random utility models choice sets of paths need to be defined. Existing approaches generate paths and assume that actual choice sets are found. On the contrary, we assume that actual choice sets are the sets of all paths connecting each origin-destination pair. These sets are however unknown and we propose a stochastic path generation algorithm that corresponds to an importance sampling approach. The path utilities should then be corrected according to the used sampling protocol in order to obtain unbiased parameter estimates. We derive such a sampling correction for the proposed algorithm.

We present numerical results based on synthetic data. The results show that the model including sampling correction yields unbiased coefficient estimates but we also make important observations concerning the Path Size attribute. Namely, it biases the estimation results if it is not computed based on the true correlation structure. These results suggest that the Path Size attribute should be computed based on as many alternatives as possible, more than in the generated choice sets.

Keywords

Route choice modeling – Path generation – Sampling of alternatives – Choice set definition

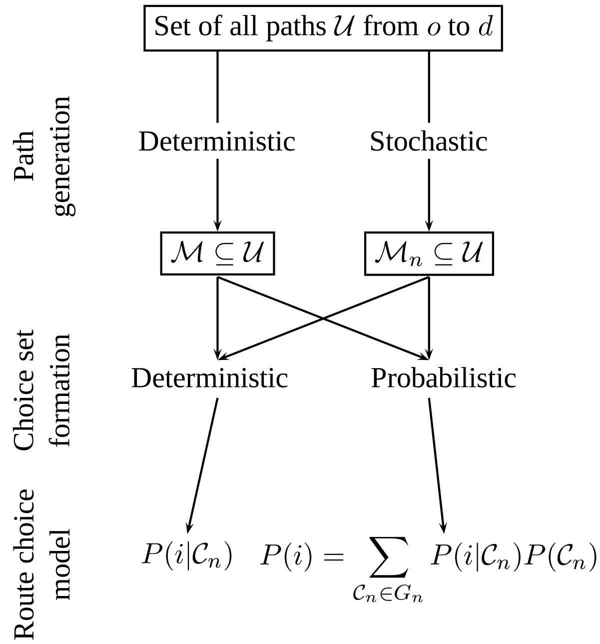

Figure 1: Route choice modeling process

1 Introduction

Route choice modeling involves several steps and we start by giving an overview of this process. Figure 1 shows schematically the different steps before arriving to the route choice model: the probability that individual n chooses path i given a choice set $\mathcal{C}_n$, $P(i|\mathcal{C}_n)$. In a real network a very large number of paths (intractable if the network contains loops) connect an origin o and destination d. This set, referred to as the universal choice set $\mathcal{U}$, is unknown. In order to estimate a route choice model a subset of paths needs to be defined and path generation algorithms are used for this purpose. There exist deterministic and stochastic approaches for generating paths. The former refers to algorithms always generating the same set $\mathcal{M}$ of paths for a given OD pair whereas an individual (or observation) specific subset $\mathcal{M}_n$ is generated with stochastic approaches. A choice set $\mathcal{C}_n$ for individual n can be defined based on $\mathcal{M}$ (or $\mathcal{M}_n$) in either a deterministic way by including all feasible paths, $\mathcal{C}_n = \mathcal{M}$ (or $\mathcal{C}_n = \mathcal{M}_n$), or by using a probabilistic model $P(\mathcal{C}_n)$ where all non-empty subsets $\mathcal{G}_n$ of $\mathcal{M}$ (or $\mathcal{M}_n$) are considered. Defining choice sets in a probabilistic way in complex due to the size of $\mathcal{G}_n$ and has never been used in a real size application. (See Manski (1977), Swait and Ben-Akiva, 1987, Ben-Akiva and Boccara, 1995, Morikawa, 1996 and Cascetta and Papola, 2001 for more details on the probabilistic approach.)

In this paper we focus on stochastic path generation and specifically on how to take into account in the route choice model that we limit the analysis to paths in $\mathcal{M}_n$. We view path generation as importance sampling of alternatives and we propose a correction of the path utilities for the sampling approach. This is a substantially different approach from existing ones because we hypothesize that the true choice set is the universal one.

In the following section we give an overview of existing path generation algorithms. An introduction to sampling of alternatives is presented in Section 3. We describe the proposed algorithm in Section 4 and we continue by deriving the sampling correction in Section 5. Numerical results based on synthetic data are presented (Section 6) before some conclusions.

2 Path Generation Algorithms

Many heuristics for generating paths have been proposed in the literature. Most of them are deterministic approaches, for example, labeled paths (Ben-Akiva et al., 1984), link elimination (Azevedo et al., 1993), link penalty (de la Barra et al., 1993), constrained k-shortest paths (e.g. van der Zijpp and Catalano, 2005) and branch-and-bound (Friedrich et al., 2001, Hoogendoorn-Lanser, 2005 and Prato and Bekhor, 2006).

Stochastic approaches are of interest for this paper since we view path generation as sampling of alternatives. Only two stochastic algorithms have been proposed in the literature. Ramming (2001) uses a simulation method that produces alternative paths by drawing link impedances from different probability distributions. The shortest path according to the randomly distributed impedance is calculated and introduced in the choice set. Recently, Bovy and Fiorenzo-Catalano (2006) proposed the so-called doubly stochastic choice set generation approach. It is similar to the simulation method but the generalized cost function has both random parameters and random attributes.

Bovy (2007) discusses the role of choice set generation in route choice modeling and gives an overview of existing approaches. Bekhor and Prato (2006) analyze empirically the effects of choice set generation on route choice model estimation results. They observe differences in the estimation results for various algorithms. They conclude that the branch-and-bound algorithm (Prato and Bekhor, 2006) performs best.

Existing approaches, both deterministic and stochastic, assume that actual choice sets are generated. Empirical studies suggest however that this is not true since in general not even all observed paths are found (see e.g. Ramming, 2001, Prato and Bekhor, 2006, Frejinger and Bierlaire, 2007 and Bierlaire and Frejinger, to appear). We assume that the true choice set is $\mathcal{U}$. This set is however to large to be enumerate and we therefore define a random sample $\mathcal{M}_n$. In order to obtain unbiased estimation results, the path utilities must be corrected according to the used sampling protocol. In the following section we give a brief introduction to sampling of alternatives.

3 Sampling of Alternatives

The multinomial logit (MNL) model can be consistently estimated on a subset of alternatives. The probability that an individual n chooses an alternative i is then conditional on the choice set $\mathcal{C}_n$ defined by the modeler. This conditional probability is

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}+\ln q(\mathcal{C}_n|i)}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}+\ln q(\mathcal{C}_n|j)}} \tag{1}$$

and includes an alternative specific term, $\ln q(\mathcal{C}_n|j)$, correcting for sampling bias. This correction term is based on the probability of sampling $\mathcal{C}_n$ given that j is the chosen alternative,

$q(\mathcal{C}_n|j)$. See for example Ben-Akiva and Lerman (1985) for a more detailed discussion on sampling of alternatives. Bierlaire, Bolduc and McFadden (2006) show that the more general family of GEV models can also be consistently estimated and propose a new estimator. Here we focus however on the MNL model.

If all alternatives have equal selection probabilities, the estimation on the subset is done in the same way as the estimation on the full set of alternatives. Namely, $q(\mathcal{C}_n|i)$ is then equal to $q(\mathcal{C}_n|j)\ \forall\ j \in \mathcal{C}_n$ (uniform conditioning property, McFadden, 1978) and the correction for sampling bias cancels out in Equation (1). This simple random sampling protocol is however not appropriate in a path generation context. First of all, we are unaware of any algorithm generating paths with equal probabilities without first enumerating all paths in $\mathcal{U}$. Second, due to the large (possibly intractable) number of paths, a simple random sample is likely to contain many alternatives that a traveler would never consider. Comparing the chosen path to a set of highly unattractive alternatives would not provide much information on the traveler's route choice. In this context, a simple random sample would need to be prohibitively large. We therefore propose a path generation algorithm that corresponds to an importance sampling approach where attractive paths have higher probability of being sampled than unattractive paths. In this case, the correction terms in Equation (1) do not cancel out and path utilities must be corrected in order to obtain unbiased results.

The Path Size Logit (PSL), proposed by Ben-Akiva and Ramming (1998) (see also Ben-Akiva and Bierlaire, 1999), and the C-Logit (Cascetta et al., 1996) models are the most commonly used MNL models for route choice analysis. An attribute, Path Size (PS) or Commonality Factor respectively, captures the correlation among paths and is added to the deterministic utilities. Up to date, these attributes are computed based on the generated choice sets. Since we assume that the true choice set is $\mathcal{U}$ we hypothesize that these attributes should be computed based on a path set as large as possible in order to approximate the true correlation structure. We test this hypothesis numerically in Section 6.

Note that existing stochastic path generation approaches may also be viewed as importance sampling approaches. It is however unclear to us how to compute the sampling correction for these algorithms.

4 A Stochastic Path Generation Approach

In this section, we first present a general stochastic approach for generating paths (also described in Bierlaire and Frejinger, 2007). The approach is flexible and can be used in various algorithms including those presented in the literature. We then describe a biased random walk algorithm that is used in this paper.

This stochastic path generation approach is based on the concept of subpath where a subpath is a sequence of links. (A link is a special case of a subpath.) We associate a probability with a subpath based on its distance to the shortest path. More precisely, its probability is defined by the double bounded Kumaraswamy distribution (Kumaraswamy, 1980) whose cumulative distribution function is $F(x_s|a,b) = 1-(1-{x_s}^a)^b$ for $x_s \in [0,1]$. a and b are shape parameters and for a given subpath s with source node v and sink node w, x_s is defined as

$$x_s = \frac{SP(o,d)}{SP(o,w)+C(s)+SP(w,d)},$$

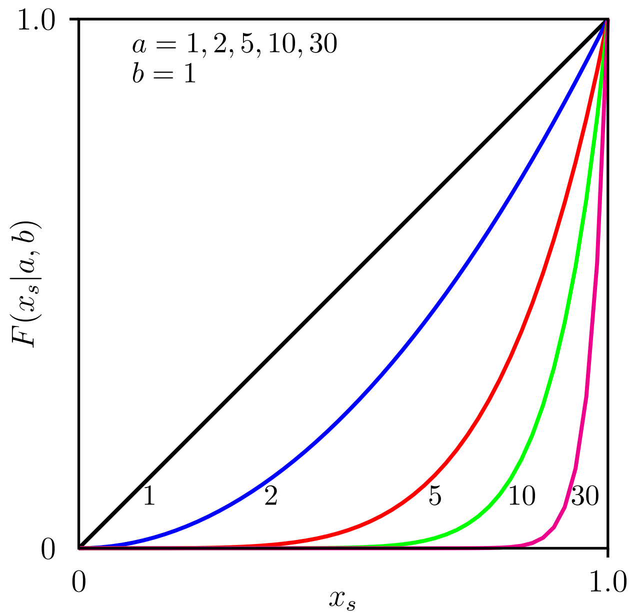

Figure 2: Kumaraswamy distribution - cumulative distribution function

where $C(s)$ is the cost of s, o the origin, d the destination and $SP(v_1, v_2)$ is the cost of the shortest path between nodes v_1 and v_2. Any generalized cost can be used in this context. Note that x_s equals one if s is part of the shortest path and $x_s \to 0$ as $C_s \to \infty$. In Figure 2 we show the cumulative distribution function for different values of a when $b = 1$. The probabilities assigned to the subpaths can be controlled by the definition of the distribution parameters. High values of a when $b = 1$ yield low probabilities for subpaths with high cost. Low values of a have the opposite effect.

This approach can be used in various algorithms. For example, in an algorithm similar to link elimination approach but where the choice of subpaths to be eliminated is stochastic. Another example is a gateway algorithm, where a subpath is selected anywhere in the network, using the probability distribution described above. A generated path is then composed of three segments: the shortest path from the origin to the source node of the subpath, the subpath itself, and the shortest path from the sink node of the subpath to the destination. This gateway algorithm was used by Bierlaire, Frejinger and Stojanovic (2006) (see also Vrtic et al., 2006) for modeling long distance route choice behavior in Switzerland.

In this paper, we use a biased random walk algorithm which has some properties which makes it appropriate as an importance sampling approach. First, it can generate potentially any path in $\mathcal{U}$. Second, path selection probabilities can be computed in a straightforward way.

4.1 Biased Random Walk Algorithm

Starting from the origin, this algorithm selects a link using the probability distribution described previously. Another link starting at the sink node of the first one is then selected and this process is applied until the destination is reached and a complete path has been generated. The algorithm biases the random walk towards the shortest path in a way controlled by the parameters of the distribution. If a uniform distribution (special case of Kumaraswamy distribution with $a = 0$ and $b = 1$) is used then the algorithm corresponds to a simple random walk. Note however that a simple random walk does not generate a simple random sample of paths.

The probability $q(j)$ of generating a path j is the probability of selecting the ordered sequence of links Γ_j

$$q(j) = \prod_{\ell \in \Gamma_j} q(\ell|\mathcal{E}_v, a, b) \tag{2}$$

where ℓ denotes a link, v its source node and $\mathcal{E}_v$ the set of outgoing links from v. In accordance with the general approach presented previously $q(\ell|\mathcal{E}_v, a, b)$ is defined by the Kumaraswamy distribution using

$$x_\ell = \frac{SP(v,d)}{C(\ell) + SP(w,d)}.$$

5 Correction for Sampling in Route Choice Models

Importance sampling takes expected choice probabilities into account, paths which are expected to have high choice probabilities have higher sampling probabilities than paths with lower expected choice probabilities. As mentioned in Section 3 the correction terms $q(\mathcal{C}_n|j)\ \forall\ j \in \mathcal{C}_n$ must be defined since they do not cancel out for this type of sampling protocol. Note however that if alternative specific constants are estimated, all parameter estimates except the constants would be unbiased even if the correction is not included in the utilities. In a route choice context it is in general not possible to estimate alternative specific constants due to the large number of alternatives and the correction for sampling is therefore essential.

We define a sampling protocol for path generation as follows: a set $\widetilde{\mathcal{C}}_n$ is generated by drawing R paths with replacement from the universal set of paths $\mathcal{U}$ and adding the chosen path to it ($|\widetilde{\mathcal{C}}_n| = R + 1$). In theory $\mathcal{U}$ can be unbounded, here we assume that paths with many loops have infinitely small sampling probability (due to the importance sampling) and we treat $\mathcal{U}$ as bounded with size J. Each path $j \in \mathcal{U}$ has sampling probability $q(j)$ and $K \sum_{j \in \mathcal{U}} q(j) = 1$ where K is a normalizing constant. (This constant does not play a role in the same way as $K_{\mathcal{C}_n}$ in Equation (5) and is therefore ignored in the following equations.)

The outcome of this protocol is $(\widetilde{k}_1, \widetilde{k}_2, \ldots, \widetilde{k}_J)$ where $\widetilde{k}_j$ is the number of times alternative j was drawn ($\sum_{j \in \mathcal{U}} \widetilde{k}_j = R$). Following Ben-Akiva (1993) we derive the formulation of $q(\mathcal{C}_n|j)$ for this sampling protocol. The probability of an outcome is given by the multinomial distribution

$$P(\widetilde{k}_1, \widetilde{k}_2, \ldots, \widetilde{k}_J) = \frac{R!}{\prod_{j \in \mathcal{U}} \widetilde{k}_j!} \prod_{j \in \mathcal{U}} q(j)^{\widetilde{k}_j}. \tag{3}$$

The number of times alternative j appears in $\widetilde{\mathcal{C}}_n$ is $k_j = \widetilde{k}_j + \delta_{jc}$, where c denotes the index of the chosen alternative and δ_{jc} equals one if $j = c$ and zero otherwise. Let $\mathcal{C}_n$ be the set containing all alternatives corresponding to the R draws ($\mathcal{C}_n = \{j \in \mathcal{U} \mid k_j > 0\}$). The size of $\mathcal{C}_n$ ranges from one to $R + 1$; $|\mathcal{C}_n| = 1$ if only duplicates of the chosen alternative were drawn and $|\mathcal{C}_n| = R + 1$ if the chosen alternative was not drawn nor were any duplicates.

Using Equation (3), the probability of drawing $\widetilde{\mathcal{C}}_n$ given the chosen alternative i can be defined as

$$q(\mathcal{C}_n|i) = q(\widetilde{\mathcal{C}}_n|i) = \frac{R!}{(k_i-1)!\prod_{\substack{j\in\mathcal{C}_n\\ j\neq i}} k_j!} q(i)^{k_i-1} \prod_{\substack{j\in\mathcal{C}_n\\ j\neq i}} q(j)^{k_j} = K_{\mathcal{C}_n} \frac{k_i}{q(i)} \tag{4}$$

where $K_{\mathcal{C}_n} = \frac{R!}{\prod_{j\in\mathcal{C}_n} k_j!} \prod_{j\in\mathcal{C}_n} q(j)^{k_j}$. We can now define the probability that an individual chooses alternative i in $\mathcal{C}_n$ as

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}+\ln\left(\frac{k_i}{q(i)}\right)}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}+\ln\left(\frac{k_j}{q(j)}\right)}}, \tag{5}$$

where $K_{\mathcal{C}_n}$ in Equation 4 does not play a role since it is constant for all alternatives in $\mathcal{C}_n$. When using the previously presented biased random walk algorithm we therefore need to count the number of times a given path j is generated as well as its sampling probability given by Equation (2).

6 Numerical Results

With the numerical results presented in this section, we want to evaluate the impact on the estimation results of

- the sampling correction;
- the definition of the PS attribute; and
- the biased random walk algorithm parameters.

We therefore use synthetic data for which the true coefficient values are known. We then evaluate different model specifications with the t-test values of the coefficient estimates with respect to (w.r.t.) their true values.

6.1 Synthetic Data

The network is shown in Figure 3 and is composed of 38 nodes and 64 links. It is a network without loops and the universal choice set $\mathcal{U}$ can therefore be enumerated ($|\mathcal{U}| = 170$). The length of the links is proportional to the length in the figure and some links have a speed bump (SB).

3000 observations have been generated by assuming a postulated model. In this case we use a PSL model, and we specify a deterministic utility function for each alternative $j \in \mathcal{U}$: $V_j = \beta_{\text{PS}}\text{PS}_j^{\mathcal{U}} + \beta_{\text{L}}\text{Length}_j + \beta_{SB}\text{SpeedBumps}_j$, where $\beta_{\text{PS}} = 1$, $\beta_{\text{L}} = -0.3$ and $\beta_{\text{SB}} = -0.1$. The PS attribute is defined by $\text{PS}_i^{\mathcal{U}} = \sum_{\ell\in\Gamma_i} \frac{L_\ell}{L_i} \frac{1}{\sum_{j\in\mathcal{U}} \delta_{\ell j}}$ where Γ_i is the set of links in path i, l_ℓ is length of link ℓ, L_i length of path i and $\delta_{\ell j}$ equals one if path j contains link ℓ, zero otherwise. Note that we explicitly index $\mathcal{U}$ since later on we compute PS based on sampled choice sets. The probability of path i is defined by $P(i|\mathcal{U}) = \frac{e^{V_i}}{\sum_{j\in\mathcal{U}} e^{V_j}}$.

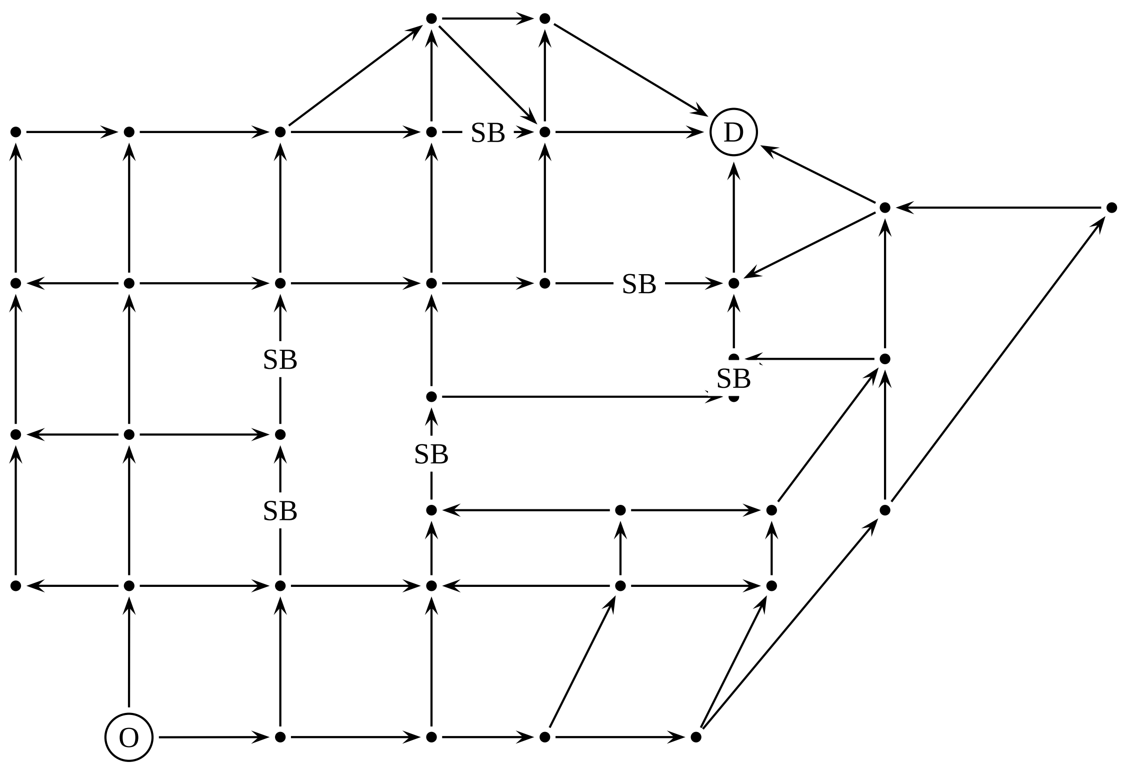

Figure 3: Example Network

6.2 Model Specifications

Four models are estimated in order to evaluate the possible combinations of with/without sampling correction and PS attribute computed based on all paths or sampled paths:

- Model $M^{\text{NoCorr}}_{PS(\mathcal{C})}$: $V_{in} = \beta_{\text{PS}}\text{PS}^{\mathcal{C}}_{in} + \beta_{\text{L}}\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i$
- Model $M^{\text{Corr}}_{PS(\mathcal{C})}$: $V_{in} = \beta_{\text{PS}}\text{PS}^{\mathcal{C}}_{in} + \beta_{\text{L}}\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i + \ln(\frac{k_i}{q(i)})$
- Model $M^{\text{NoCorr}}_{PS(\mathcal{U})}$: $V_i = \beta_{\text{PS}}\text{PS}^{\mathcal{U}}_i + \beta_{\text{L}}\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i$
- Model $M^{\text{Corr}}_{PS(\mathcal{U})}$: $V_j = \beta_{\text{PS}}\text{PS}^{\mathcal{U}}_i + \beta_{\text{L}}\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i + \ln(\frac{k_i}{q(i)})$

The PS attribute based on sampled paths is defined by $\text{PS}^{\mathcal{C}}_{in} = \sum_{\ell\in\Gamma_i} \frac{l_\ell}{L_i} \frac{1}{\sum_{j\in\mathcal{C}_n}\delta_{\ell j}}$.

6.3 Estimation Results

Table 1 shows estimation results [1] for a specific parameter setting of the biased random walk algorithm (10 draws, Kumaraswamy parameters $a = 5$ and $b = 1$). Length is used as generalized cost for the shortest path computations.

[1]There may be an issue with the scale for the computation of the t-tests in models $M^{\text{NoCorr}}_{PS(\mathcal{C})}$ and $M^{\text{Corr}}_{PS(\mathcal{C})}$. This will be further investigated but we do not anticipate that this changes the conclusions of the results.

	True PSL	$M^{\text{NoCorr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{NoCorr}}_{PS(\mathcal{U})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{U})}$ PSL
$\widehat{\beta}_{\text{PS}}$	**1**	**0.363**	**0.443**	**-0.203**	**1.03**
Standard error		0.0729	0.086	0.0487	0.0465
t-test w.r.t. 1		-8.74	-6.48	-24.70	0.65
$\widehat{\beta}_L$	**-0.3**	**-0.0529**	**-0.326**	**-0.0453**	**-0.291**
Standard error		0.00864	0.0085	0.00828	0.00788
t-test w.r.t. -0.3		28.60	-3.06	30.76	1.14
$\widehat{\beta}_{\text{SB}}$	**-0.1**	**-0.345**	**-0.134**	**-0.404**	**-0.0773**
Standard error		0.0315	0.0259	0.0298	0.0258
t-test w.r.t. -0.1		-7.78	-1.31	-10.20	0.88
Final Log-likelihood		-6596.22	-6047.14	-6598.46	-5840.80
Adj. rho square		0.02	0.10	0.02	0.13
Null Log-likelihood: -6719.733, 3000 observations Algorithm parameters: 10 draws, $a = 5$, $b = 1$, $C(\ell) = L_\ell$ Average size of sampled choice sets: 9.43 BIOGEME (Bierlaire, 2007, Bierlaire, 2003) has been used for all model estimations					

Table 1: Path Size Logit Estimation Results

Our hypothesis in Section 3 stating that the PS attribute should be computed based on $\mathcal{U}$ and not $\mathcal{C}_n$ is confirmed by these results. Indeed, the two models where PS is based on $\mathcal{C}_n$ ($M^{\text{NoCorr}}_{PS(\mathcal{C})}$ and $M^{\text{Corr}}_{PS(\mathcal{C})}$) have biased coefficient estimates. Note however that the model including a sampling correction, $M^{\text{Corr}}_{PS(\mathcal{C})}$, has coefficient estimates closer to their true values than the one which has not, $M^{\text{NoCorr}}_{PS(\mathcal{C})}$.

The model including sampling correction and PS based on $\mathcal{U}$ ($M^{\text{Corr}}_{PS(\mathcal{U})}$) has unbiased coefficient estimates and has a remarkably better model fit than the other models. In $M^{\text{NoCorr}}_{PS(\mathcal{U})}$, even though the PS attribute is based on $\mathcal{U}$ the results are completely biased due to the lacking sampling correction. This clearly shows the strength of the sampling approach proposed here.

We now analyze the estimation results as a function of two of the biased random walk algorithm parameters: the Kumaraswamy distribution parameter a and the number of draws. Figure 4 shows the absolute value of the t-tests with respect to the true values for model $M^{\text{Corr}}_{PS(\mathcal{U})}$. Independently of the algorithm parameters, $\widehat{\beta}_{\text{PS}}$ and $\widehat{\beta}_{\text{SB}}$ are unbiased. As expected, the results deteriorate as a increase. This is particularly the case for $\widehat{\beta}_L$ that is significantly different from its true value for $a > 20$. Recall from Figure 2 that the higher the value of a the more the biased random walk is oriented towards the shortest path. Finally we note that the results are rather constant for different number of draws. In the Appendix we present the results for all four models in table form. The other three models do not have unbiased results for all three coefficients for any of the parameter settings. This clearly shows the superiority of the approach with sampling correction and computing PS based on the true correlation structure.

Finally we show in Figure 5 the average number of paths in the choice sets. As expected, the number of paths increase with the number of draws but decrease as a increase. Based on the estimation results we can however conclude that unbiased results can be obtained with rather

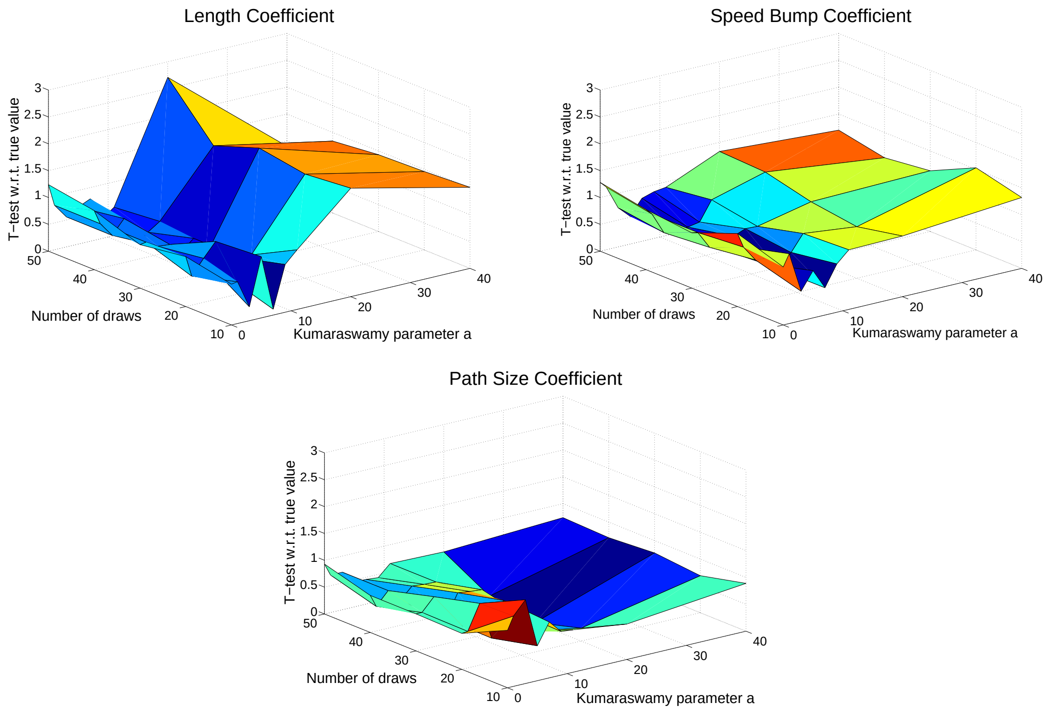

Figure 4: T-test values with respect to true values for the coefficients of $M^{\text{Corr}}_{PS(\mathcal{U})}$

few draws.

7 Conclusions and Future Work

This paper presents a substantially different approach for choice set generation and route choice modeling compared to existing ones. We view path generation as an importance sampling approach and derive a sampling correction to be added to the path utilities. We hypothesize that the true choice set is the set of all paths connecting an origin destination pair. Accordingly, we propose to compute the PS attribute on all path (or as many as possible) so that it reflects the actual correlation structure.

We present numerical results based on synthetic data which clearly show the strength of the approach. Models including a sampling correction are remarkably better than the ones that do not. Moreover, unbiased estimation results are obtained if the PS attribute is computed based on all paths and not on generated choice sets. This is a completely different approach from route choice modeling praxis where generated choice sets are assumed to correspond to the true ones and PS (or Commonality Factors) is computed on these generated path sets.

In the near future we will continue the analysis of the number of paths used for the Path Size computation. We will also test the approach on real GPS data set from Sweden.

8 Acknowledgments

We have benefited from discussions with Moshe Ben-Akiva, Piet Bovy and Mogens Fosgerau.

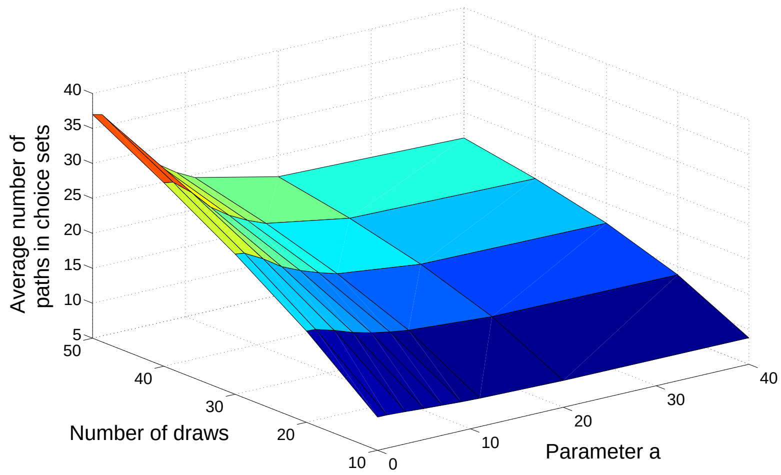

Figure 5: Average number of paths in choice sets

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A Additional Estimation Results

The following tables show the absolute value of t-test values for the four different models discussed in the paper.

		Kumaraswamy parameter a								
Coef.	Nb. draws	0	1	3	5	7	9	11	20	40
	10	4.47	3.80	18.13	30.76	40.85	49.38	55.29	50.81	10.57
	20	4.21	3.60	18.79	30.76	40.30	49.23	57.46	65.00	17.88
$\widehat{\beta}_L$	30	4.15	4.04	18.57	30.11	39.70	48.01	56.61	69.91	21.57
	40	3.54	4.18	18.38	29.51	39.06	47.52	55.10	74.19	25.73
	50	3.18	4.32	18.04	29.13	38.45	46.28	53.92	76.50	28.50
	10	20.71	17.44	13.39	10.20	8.55	7.45	7.22	6.42	5.91
	20	21.20	17.70	12.50	8.90	7.06	6.25	5.74	5.91	4.96
$\widehat{\beta}_{\text{SB}}$	30	20.96	17.09	11.37	7.54	5.52	4.69	4.15	4.61	4.46
	40	20.03	16.33	10.35	6.35	4.45	3.43	3.11	3.66	3.84
	50	19.23	15.35	9.43	5.57	3.24	2.36	1.99	2.84	3.24
	10	33.73	31.28	26.34	24.70	24.13	24.00	23.84	20.70	14.26
	20	31.97	28.91	23.58	21.93	21.35	21.35	21.28	19.76	11.18
$\widehat{\beta}_{\text{PS}}$	30	29.55	25.61	20.96	19.71	19.22	19.25	19.50	18.57	8.96
	40	26.84	23.12	18.69	17.91	17.70	18.02	18.30	17.87	7.66
	50	24.69	20.90	16.88	16.45	16.53	16.93	17.36	17.30	6.50

Table 2: Model $M_{PS(\mathcal{U})}^{\text{NoCorr}}$

Coef.	Nb. draws	Kumaraswamy parameter a 0	1	3	5	7	9	11	20	40
$\widehat{\beta}_L$	10	1.10	6.98	17.74	28.60	36.66	44.12	47.57	48.08	10.43
	20	0.98	6.19	18.04	27.97	35.39	41.64	47.04	51.91	17.93
	30	0.24	6.26	17.61	26.73	32.61	39.70	44.31	51.15	21.46
	40	0.74	5.88	17.21	26.08	33.21	38.67	43.06	50.62	25.50
	50	0.25	5.93	16.84	25.98	32.67	37.08	41.24	50.38	28.10
$\widehat{\beta}_{\text{SB}}$	10	15.74	13.39	10.80	7.78	6.31	6.05	5.46	5.86	5.65
	20	14.32	12.27	8.95	6.24	5.00	4.55	5.32	5.85	5.24
	30	13.84	10.81	7.25	4.38	0.43	3.25	3.04	5.25	4.69
	40	11.69	10.35	6.11	3.26	2.63	2.04	2.57	4.40	3.86
	50	11.68	8.73	5.18	3.46	1.67	1.23	1.40	3.58	2.64
$\widehat{\beta}_{\text{PS}}$	10	4.39	5.43	7.64	8.74	9.50	12.26	10.85	10.78	8.86
	20	5.46	6.96	8.65	10.15	11.84	12.88	15.52	13.36	11.61
	30	7.29	6.89	8.49	9.20	9.05	13.28	14.01	15.21	11.99
	40	4.94	8.04	8.27	9.42	12.23	13.31	15.31	15.83	11.44
	50	6.83	6.25	8.13	11.14	12.45	13.50	15.19	16.21	10.21

Table 3: Model $M_{PS(\mathcal{C})}^{\text{NoCorr}}$

Coef.	Nb. draws	Kumaraswamy parameter a 0	1	3	5	7	9	11	20	40
$\widehat{\beta}_L$	10	4.24	3.87	4.77	3.06	2.78	0.36	0.15	2.45	1.68
	20	2.83	3.52	4.73	4.60	4.22	3.06	1.20	0.61	1.47
	30	2.51	2.79	5.15	6.62	6.96	4.58	4.44	0.34	0.98
	40	1.51	2.52	5.32	7.32	6.44	5.37	4.32	0.78	0.17
	50	1.25	2.13	5.36	7.38	6.57	5.98	4.86	0.79	0.92
$\widehat{\beta}_{\text{SB}}$	10	2.46	1.59	2.00	1.31	0.96	0.46	0.73	1.86	2.48
	20	0.34	0.62	0.62	0.40	0.60	0.43	0.46	2.28	3.21
	30	0.92	2.50	2.41	1.24	1.90	1.13	1.66	2.53	3.28
	40	3.71	3.17	3.45	2.17	0.90	1.10	1.61	2.59	3.53
	50	3.85	5.18	4.28	1.82	1.06	1.45	1.60	2.59	4.07
$\widehat{\beta}_{\text{PS}}$	10	5.74	5.15	5.44	6.48	6.25	7.62	5.38	5.18	5.21
	20	6.11	6.00	5.42	6.82	7.30	6.99	8.11	5.14	5.69
	30	7.13	5.17	4.18	4.43	3.47	6.09	5.40	5.11	5.03
	40	4.20	5.61	3.14	3.34	5.71	5.73	6.03	5.90	4.31
	50	5.17	3.34	2.24	4.05	5.60	5.56	6.07	5.89	3.63

Table 4: Model $M_{PS(\mathcal{C})}^{\text{Corr}}$

Coef.	Nb. draws	Kumaraswamy parameter a								
		0	1	3	5	7	9	11	20	40
	10	0.76	0.78	0.26	1.14	0.11	0.89	1.11	2.02	1.51
	20	0.93	0.54	0.82	1.08	0.37	0.67	0.67	1.94	1.46
$\widehat{\beta}_L$	30	0.81	0.83	0.70	0.68	0.50	0.69	0.30	2.08	1.43
	40	1.10	0.83	0.56	0.82	0.52	0.46	0.61	1.78	1.34
	50	1.24	0.83	0.56	0.69	0.79	0.36	0.53	2.71	0.90
	10	1.08	1.30	0.55	0.88	0.51	0.90	1.11	1.13	1.32
	20	1.36	1.08	0.79	0.65	0.50	0.77	1.07	0.96	1.53
$\widehat{\beta}_{\text{SB}}$	30	1.03	1.01	0.68	0.59	0.78	0.87	0.86	1.08	1.13
	40	1.12	1.01	0.61	0.57	0.61	0.65	1.01	1.29	1.03
	50	1.28	1.17	0.73	0.52	0.80	0.87	0.90	1.33	1.20
	10	1.07	1.31	1.56	0.65	1.04	0.81	0.79	0.66	0.89
	20	0.67	0.68	1.15	0.45	1.10	0.84	0.42	0.24	0.69
$\widehat{\beta}_{\text{PS}}$	30	0.69	0.69	0.93	0.46	0.90	0.87	0.85	0.00	0.77
	40	0.69	0.46	0.70	0.46	0.68	0.66	0.65	0.15	0.71
	50	0.93	0.70	0.70	0.47	0.46	0.44	0.65	0.63	0.74

Table 5: Model $M_{PS(\mathcal{U})}^{\text{Corr}}$